KRISHNA MURTHY IIT ACADEMY

ASSIGNMENT-7-SOLUTIONS

VECTORS AND 3D - SOLUTIONS

1. $\overline{OQ}\times\overline{OR}=\begin{vmatrix} i & j & k \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix}=-10i+10j-5k=\overline{n}$

Req.Dist. = Projection of $\overline{PO}$ on $\overline{OQ}\times\overline{OR}$

$=\dfrac{\overline{OP}.\overline{n}}{|\overline{n}|}=\dfrac{45}{15}=3$

2. Let the required vector be $\overline{d}=\ell\overline{b}+m\overline{c}=(\ell+m)i+(2\ell+m)j-(\ell+2m)k$

Projection of $\overline{d}$ along $\overline{a}=\dfrac{\overline{d}.\overline{a}}{|\overline{a}|}$

$\Rightarrow\sqrt{\dfrac{2}{3}}=\dfrac{2(\ell+m)-(2\ell+m)-(\ell+m)}{\sqrt{6}}\Rightarrow\ell=-m-2$

$\therefore\overline{d}=(-m-2)(i+2j-k)+m(i+j-2k)$

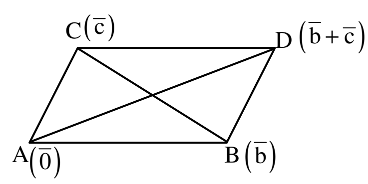
$=-2i-(m+4)j+(2-m)k$

We cannot find m.

3. $\overline{AD}^2-\overline{BC}^2=(\overline{b}+\overline{c})^2-(\overline{c}-\overline{b})^2=4\overline{b}.\overline{c}=4\overline{AB}.\overline{AC}$

$=4AB(\text{Projection of }\overline{AC}\text{ on }\overline{AB})$

$=4$ rectangle contained by $\overline{AB}$ and projection of $\overline{AC}$ on $\overline{AB}$



4. $\overline{p}.\overline{a}=\left[\overline{b}-(\overline{a}.\overline{b})a\right].\overline{a}=\overline{b}.\overline{a}-\overline{a}.\overline{b}(\overline{a}.\overline{a})=0$

$\overline{p}.\overline{p}=|\overline{p}|^2=1-(\overline{a}.\overline{b})^2=1-\cos^2 t=\sin^2 t\ \because\overline{a}.\overline{b}=\cos\gamma$

$\overline{p}.\overline{q}=|\overline{p}||\overline{q}|\cos\theta=\sin\gamma\sin\beta\cos\theta(\because|\overline{q}|=\sin\beta)$

Again $\overline{p}.\overline{q}=\left[\overline{b}-(\overline{a}.\overline{b})a\right].\left[\overline{c}-(\overline{a}.\overline{c})\overline{a}\right]=\cos\alpha-\cos\beta\cos\gamma$

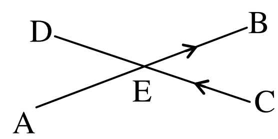
5. $\overline{AB}=m(2i-5j+2k),\overline{BC}=(i-2j)$

$\overline{CD}=n(-6i+15j-3k)$

If $\overline{AB}$ and $\overline{CD}$ intersect at E, then

$\overline{EB}=p\overline{AB}$ and $\overline{CE}=a\overline{CD}$

But, $\overline{EB}+\overline{BC}+\overline{CE}=\overline{0}\Rightarrow p\overline{AB}+q\overline{CD}+\overline{BC}=0$



$\Rightarrow pm(2i-6j+2k)+(i-2j)+qn(-6i+15j-3k=0)$

$\Rightarrow 2pm+1-6qn=0,-6pm-2+15qn=0,2pm-3qn=0$

Solving for pm and qn, we get $pm=\dfrac{1}{2},qn=\dfrac{1}{3}$

$\Rightarrow p=\dfrac{1}{2m},q=\dfrac{1}{3n}\Rightarrow 0<\dfrac{1}{2m}\le 1,0\le\dfrac{1}{3n}\le 1$

$\Rightarrow m\ge\dfrac{1}{2},n\ge\dfrac{1}{3}$. Area of $\Delta BCE=\dfrac{1}{2}\left|\overline{EC}\times\overline{EB}\right|$

$=\dfrac{1}{2}pqnm\begin{vmatrix} i & j & k \\ -6 & 15 & -3 \\ 2 & -6 & 2 \end{vmatrix}=\dfrac{1}{2}.\dfrac{1}{2}.\dfrac{1}{3}|12i-6j-6k|=\dfrac{\sqrt{6}}{2}$

6. $\overline{a}\times\overline{b}=2i+2j+k, \overline{a}+\overline{b}=i-3j-4k$

Now, let $\overline{a}=xi+yj+zk$ then

$(xi+yj+zk)\times(i-3j-4k)=2i+2j+k$

$\begin{vmatrix} i & j & k \\ x & y & z \\ 1 & -3 & -4k \end{vmatrix}=2i+2j+k$, equate coefficients of i, j, k and solve to get $x=1, y=-2, z=-2$

7. $a\times b=2i+2j-k$

$a\times b=i-3j+4k$

$a=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$

$b=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$

$a*b=(x_1+x_2)\hat{i}+(y_1+y_2)\hat{j}+(z_1+z_2)\hat{k}$

$x_1+x_2=1, y_1+y_2=-3, z_1+z_2=4$

$(a\times b)=(x\hat{i}+y\hat{j}+z\hat{k})\times(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})$

$=i(y_1z_2-y_2z_1)+j(x_2z_1-z_2x_1)+k(x_1y_2-x_2y_1)$

$y_1z_2-y_2z_1=2; \qquad x_1y_2-x_2y_1=-1$

$x_2z_1-z_2x_1=2$

$x_2=(1-x_1), y_2=(3+y_1); z_2=4-z_1$

$(1-x_1)z_1-(4-z_1)x_1=2$

$z_1-z_1x_1-4x_1+x_1z_1=2\Rightarrow z_1=2+4x_1$

$z_2=2-4x_1$

$y_1z_2-y_2z_1=2$

$y_1(2-4x_1)+(3+y_1)(2+4x_1)=2$

$2y_1-4x_1y_1+6+2y_1+4x_1y_1+12x_1=2$

$4y_1=-4-12x_1$

$y_1=-(1+3x_1)\Rightarrow y_2=-(3-1-3x_1)=-(2-3x_1)$

$x_2y_1-x_1y_2=1$

$(1-x_1)(-)(1+3x_1)-x_1(2-3x_1)=1$

$-(-3x_1^2+3x_1+1-x_1)-2x_1+3x_1^2=1$

$-(1+2x_1-3x_1^2)-2x_1+3x_1^2=1$

$-1-2x_1+3x_1^2-2x_1+3x_1^2=1$

$3x_1^2-2x_1-1=0$

$x_1=\dfrac{2\pm\sqrt{4+12}}{6}=\dfrac{2\pm\sqrt{16}}{6}=\dfrac{2\pm4}{6}$

$x_1=1;-\dfrac{1}{3},\ x_2=0;\dfrac{4}{3}$

$y_1=-(4),0;\ y_2=+1,-3$

8. Given mod of $\begin{vmatrix} x+a & x+b & x+c \\ x+b & x+c & x+d \\ x+a-c & x-1 & x-b+d \end{vmatrix}=2$

Apply $C_1\to C_1+C_3-2C_2$, and use that a, b, c are in A.P. to get

$\begin{vmatrix} 0 & x+b & x+c \\ 0 & x+c & x+d \\ 2 & x-1 & x-b+8 \end{vmatrix}=2\Rightarrow\left|bd-c^2\right|=1$

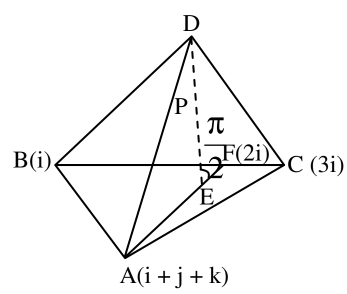
$\Rightarrow\left|(c-d)(c+d)-c^2\right|=1\Rightarrow d=\pm1$

9. $\begin{vmatrix} 1+\sin^2 A & \sin^2 A & \sin^2 A \\ \cos^2 A & 1+\cos^2 A & \cos^2 A \\ 4\sin 2A & 4\sin 2A & 1+4\sin 2A \end{vmatrix} = 0$

$$\left(1+\sin^2 A\right)\left(\left(1+\cos^2 A\right)\left(1+4\sin 2A\right)-4\sin 2A\cos^2 A\right)+\sin^2 A\left(4\sin 2A\cos^2 A-\cos^2 A-4\sin 2a\cos^2 A\right)$$
$$+\sin^2 A\left[4\sin 2a\cos^2 A-4\sin 2A-4\sin 2A\cos^2 A\right]=0$$
$$\left(1+\sin^2 a\right)\left[1+\cos^2 A+4\sin 2A\right]+\sin^2 A\left[-\cos^2 A-4\sin 2A\right]+1+\cos^2 A+4\sin 2A$$
$$+\sin^2 A\left[1+\cos^2 A+4\sin A-\cos^2 A-4\sin 2A\right]$$
$$2+4\sin 2A=0\Rightarrow \sin 2A=-\frac{1}{2}$$
$$2A=(2n+1)\pi+(-1)^n\frac{\pi}{6} \text{ or } A=(2n+1)\frac{\pi}{2}+(-1)^n\frac{\pi}{12}$$

10. Given AD = 4, Volume $=\frac{2\sqrt{2}}{3}$

Let DE = P, then



$$\frac{1}{3}\times \text{Area of } \Delta ABC.p=\frac{2\sqrt{2}}{3}$$
$$\Rightarrow \frac{1}{2}\left|\overline{BA}\times\overline{BC}\right|p=2\sqrt{2}$$
$$\Rightarrow \frac{1}{2}\left|(j+k)\times 2i\right|p=2\sqrt{2}\Rightarrow p=2$$

Let E divides median AF in the ratio $\lambda:1$ then, p.v. of E = $\dfrac{\lambda.2i+(i+j+k)}{\lambda+1}$

$$\therefore \overline{AE}=\text{p.v. of E}-\text{p.v. of A}=(i-j-k)\frac{\lambda}{\lambda+1}$$
$$p^2+AE^2=AD^2\Rightarrow 4+\left(\frac{\lambda}{\lambda+1}\right)^2-3=16\Rightarrow \lambda=-2 \text{ or } \frac{-2}{3}$$
$$\therefore \text{p.v. of E is } -i+3j+3k \text{ or } 3i-j-k$$

11. $x\times y=a;\ y\times z=b;\ x.b=c;\ x.y=1=y.z$

$$(x\times y)\times b=a\times b$$
$$\Rightarrow (x.b)\overline{y}-(y.b)\overline{x}=a\times b$$
$$\Rightarrow c\overline{y}=a\times b\Rightarrow \overline{y}=\frac{a\times b}{c}$$

Now, $(x\times y)\times y=a\times y$

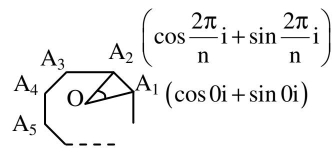
$$(x.y)\overline{y}-\left(y^2\right)\overline{x}=a\times y=a\times\frac{(a\times b)}{c}$$
$$\frac{(a\times b)}{c}-\frac{|a\times b|^2}{c^2}\overline{x}=\frac{(a.b)\overline{a}}{c}-\frac{a^2 b}{c}$$
$$\frac{(a\times b)}{c}-\frac{(a.b)\overline{a}}{c}-\frac{\left|a^2\right|b}{c}=\frac{x|a\times b|^2}{c^2}$$
$$x=\frac{c}{|a\times b|^2}\left[|a|^2 b-(a.b)\overline{a}+(a\times b)\right]$$
$$y\times z=b\Rightarrow y^2 z-(y.z)\overline{y}=b\times\frac{(a\times b)}{c}$$
$$\frac{(a\times b)^2}{c^2}z-\frac{(a\times b)}{c}=\frac{b^2\overline{a}}{c}-\frac{(a.b)}{c}\overline{b}$$
$$\frac{|a\times b|^2}{c^2}z=\frac{1}{c}\left[b^2\overline{a}-(a.b)\overline{b}+(a\times b)\right]$$

$$\Rightarrow z = \frac{c}{|a \times b|^2} 2\left[\left(b^2\right)\bar{a} - (a.b)\bar{b} + (a \times b)\right]$$

12. i) Consider a 'n' sided polygon (regular) of circum radius 1 unit.

Let O be origin and the vertices be $a_1, a_2, a_3, a_4, ...a_n$ such that

$|a_1| = |a_2| = ... = 1$



Consider $OA_1.OA_2 + OA_1.OA_3 + OA_1.OA_4 + ...\overline{OA_1}.\overline{OA_n}$

$$= 1\left(\cos\frac{2\pi}{n} + \cos\frac{4\pi}{n} + ... + ...\cos\frac{2(n-1)}{n}\pi\right)$$

$$\sum_{r=1}^{n-1} \overline{OA_r} \times \overline{OA_{r+1}} = (1-n)\left(\overline{OA_2} \times \overline{OA_1}\right)$$

ii) Let the p.v. of B and C be $\bar{b}$ and $\bar{c}$ referred to A as origin then,

p.v.of D is $\frac{n\bar{c} + \bar{b}}{n+1}$

p.v.of E is $\frac{\bar{c}}{n+1}$

p.v. of F is $\frac{n\bar{b}}{n+1}$

$$\overline{DE} = \overline{AE} - \overline{AD} = \frac{(1-n)\bar{c} - \bar{b}}{n+1}$$

$$\overline{DF} = \overline{AF} - \overline{AD} = \frac{(n+1)\bar{b} - n\bar{c}}{n+1}$$

Vector area of $\Delta ABC = \frac{1}{2}\left(\overline{AB} \times \overline{BC}\right) = \frac{1}{2}\bar{b} \times \bar{c}$

Vector are of $\Delta DEF = \frac{1}{2}\left(\overline{DE} \times \overline{DF}\right) = \frac{n^2 - n + 1}{(n+1)^2} = \frac{1}{2}\bar{b} \times \bar{c}$

13. $\cos A i + j + k; i + \cos B j + k; i + j + \cos C k$

$$\begin{vmatrix} \cos A & 1 & 1 \\ 1 & \cos B & 1 \\ 1 & 1 & \cos C \end{vmatrix} = 0$$

$$\Rightarrow \pi \cos A + 2 = \sum \cos A$$

$$\left(1 - 2\sin^2\frac{A}{2}\right)\left(1 - 2\sin^2\frac{B}{2}\right)\left(1 - 2\sin^2\frac{C}{2}\right) + 2 = 3 - 2\sum \sin^2\frac{A}{2}$$

$$3 - 2\sum \sin^2\frac{A}{2} + 4\sum \sin^2\frac{A}{2}\sin^2\frac{B}{2} - 8\pi \sin^2\frac{A}{2} = 3 - 2\sum \sin^2\frac{A}{2}$$

$$\Rightarrow \sum \cos^2\frac{A}{2} = \frac{8}{4} = 2$$

14. Consider

$\bar{p} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\bar{q} = x\hat{i} + y\hat{j} + z\hat{k}$

$|\bar{p}| = 5; \quad |\bar{q}| = 6$

$p.q = 30$

$\Rightarrow \cos\theta = 0$

$\Rightarrow$ collinear vectors

$(a + b + c) = t(x + y + z)$

$\left(a\hat{i} + b\hat{j} + c\hat{k}\right) = t\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

$\Rightarrow t = \frac{5}{6}$

$$\Rightarrow \frac{a + b + c}{x + y + z} = \frac{5}{6}$$

15. $|a|=2, |b|=3$

$[a\ b\ c]=2$

$a+c\times b+c=0$

$b\times c=a+c$

$2=a^2+a.c$

$a.c=-2$

$c.(b\times c)=(a+c).c$

$a.c+c^2=0 \Rightarrow c^2=-a.c=2$

$|\bar{c}|=\sqrt{2}$

$a\times(b\times c)=a\times(a+c)$

$(a.c)\bar{b}-(a.b)\bar{c}=a\times c$

$(a.c)\bar{b}=a\times c$

$2\bar{b}=\bar{c}\times\bar{a}$

$|\bar{a}\times\bar{c}|=2|\bar{b}|=2\times 3=6$

For a, b, c to be an orthonormal triad

$a\times c=|a||c|$

$|c||a|=6$

$|c|=\frac{b}{|a|}3$

16. Take the mutual perpendicular planes as coordinate planes. Let the equation of the plane be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$……………..(1)

It meets the coordinate axes at $A(a,0,0), B(0,b,0), C(0,0,c)$ which are the vertices of triangle ABC.

The d.rs. of sides AB and AC are $0-a, b-0, 0-0$ and $0-a, 0-0, c-0$. So the angle A between them is given by

$$\tan A=\sqrt{\frac{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_2b_2-a_2b_1)^2}{a_1a_2+b_1b_2+c_1c_2}}$$

$$\Rightarrow \cot A=\frac{a^2}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}$$

Similarly, $\cot B=\frac{b^2}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}$ and $\cot C=\frac{c^2}{\sqrt{b^2c^2+c^2a^2+a^2b^2}}$

Let the plane (1) makes an angle α with yz-plane i.e. $x=0$. Since the direction ratios of normals to these planes are $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ and $1,0,0$ respectively $\cos\alpha=\frac{\frac{1}{a}.1+\frac{1}{b}.0+\frac{1}{c}.0}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}.\sqrt{1}}$

$\Rightarrow \cos^2\alpha=\cot B.\cot C$

Similarly, $\cos^2\beta=\cot C\cot A$ and $\cos^2\gamma=\cot A\cot B$

17. Equations of the angular bisectors of the lines $\frac{x}{\ell_1}=\frac{y}{m_1}=\frac{z}{n_1}$ and $\frac{x}{\ell_2}=\frac{y}{m_2}=\frac{z}{n_2}$ are given by

$$\frac{x}{\ell_1+\ell_2}=\frac{y}{m_1+m_2}=\frac{z}{n_1+n_2} \text{ and } \frac{x}{\ell_1-\ell_2}=\frac{y}{m_1-m_2}=\frac{z}{n_1-n_2}$$

Then find the plane containing the above two lines. Let the plane be $ax+by+cz+d=0$. Since it contains the lines $a(\ell_1+\ell_2)+b(m_1+m_2)+c(n_1+n_2)=0$ and $a.0+b.0+c.0+d=0 \Rightarrow d=0$

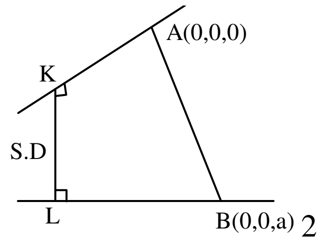
Similarly other line.

$\therefore a(\ell_1\pm\ell_2)+b(m_1\pm m_2)+c(n_1\pm n_2)=0$. Proceed.

18. The pairs of opposite edges are $y+z=0, x+z=0, x+y=0$ and $x+y+z=0$. They can be written as

$\frac{x}{1}=\frac{y}{1}=\frac{z}{-1}$ and $\frac{x}{1}=\frac{y}{-1}=\frac{z-a}{0}$

Let ℓ, m, n be the direction cosines of KL (the line of SD).
But, SD is $\perp$ar to both the lines



$\therefore \ell+m-n=0; \ell-m+0=0$

$$\Rightarrow \frac{\ell}{1}=\frac{m}{1}=\frac{n}{2}=\frac{\sqrt{\ell^2+m^2+n^2}}{\sqrt{1+1+4}}$$

$\Rightarrow$ direction cosines of S.D. are $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

Let $A(0,0,0), B(0,0,a)$ be two points on the lines then length of S.D. = Projection AB on KL

$$=\frac{1}{\sqrt{6}}(0-0)+\frac{1}{\sqrt{6}}(0-0)+\frac{2}{\sqrt{6}}(a-0)=\frac{2a}{\sqrt{6}}$$

The equations of KL, the line of intersection of the planes AKL and BKL are

$$\begin{vmatrix} x & y & z \\ 1 & 1 & -1 \\ 1 & 1 & 2 \end{vmatrix}=0 \text{ and } \begin{vmatrix} x & y & z-a \\ 1 & -1 & 0 \\ 1 & 1 & 2 \end{vmatrix}=0$$

Or

$3x-3y=0$ and $-2x-2y+2z-2a=0 \Rightarrow x-y=0$ and $x+y-z+a=0$ which clearly passes through $x=y=z=-a$ which is $(-a,-a,-a)$

19. All given lines are passing through origin 'O'. Thus the lines will be coplanar if they are perpendicular to a line through 'O'. Let ℓ, m, n be d.c.'s of this line through O, then

$a\alpha\ell+b\beta m+c\gamma m=0$ ....(1)

$\left(\frac{\alpha}{a}\right)\ell+\left(\frac{\beta}{b}\right)m+\left(\frac{\gamma}{c}\right)n=0$ ....(2)

And $\alpha\ell+\beta m+\gamma n=0$ ....(3)

Eliminating $\alpha\ell, \beta m+\gamma n$ we get,

$$\begin{vmatrix} a & b & c \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \\ 1 & 1 & 1 \end{vmatrix}=0$$ apply C_2-C_1, C_3-C_1 and expand to get $(a-b)(b-c)(c-a)=0$

20. Image of a point (x_1, y_1, z_1) w.r.to the line through (a,b,c) with direction cosines ℓ, m, n is

$(2\ell r-x_1, 2mr-y_1, 2nr-z_1)$ where $r=\frac{\ell(x_1-a)+m(y_1-b)+n(z_1-c)}{\ell^2+m^2+n^2}$

Taken any three points on the plane, $2x+3y+z=1$ find their images and find equation of plane through the three points. Here $(a,b,c)=(0,0,0)$

$$(\ell, m, n)=\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

Let $(x_1, y_1, z_1)=(0,0,1)$, then $r=\frac{3}{\sqrt{14}}$

Image is $\left(\frac{6}{14}-0, \frac{12}{14}-0, \frac{18}{14}-1\right)=\left(\frac{3}{7}, \frac{6}{7}, \frac{2}{7}\right)$

Similarly take two more points and complete.

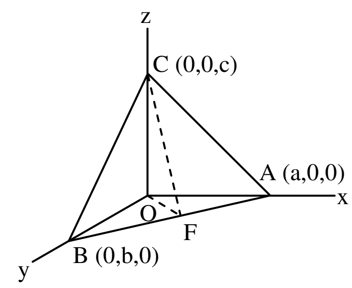
21. Any point on given line is $A(5r-6, 3r-10, 8r-14)$. Let the given point be $P(7,2,4)$. The direction ratios of AP are $5r-13, 3r-12, 8r-18$. Angle between AP and given line is

$$\cos 45°=\frac{5(5r-13)+3(3r-12)+8(8r-18)}{\sqrt{5^2+3^2+8^2}.\sqrt{(5r-13)^2+(3r-12)^2+(8r-18)^2}}$$

Solve the quadratic in r to set two values for r and hence two lines.

$$\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2} \text{ and } \frac{x-7}{2}=\frac{y-2}{-8}=\frac{z-4}{6}$$

22. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes at A, B, C. Prove that the planes through the axes and the internal bisectors of the angles A, B, C pass through the line $\frac{x}{a\sqrt{b^2+c^2}}=\frac{y}{b\sqrt{c^2+a^2}}=\frac{z}{c\sqrt{a^2+b^2}}$. Also find the incentre of ΔABC.



Let CF be the internal bisector of angle ACB then $\frac{BF}{AF}=\frac{BC}{AC}=\frac{\sqrt{b^2+c^2}}{\sqrt{c^2+a^2}}$

$\therefore$ F is $\left(\frac{a\sqrt{b^2+c^2}}{\sqrt{b^2+c^2}+\sqrt{c^2+a^2}},\frac{b\sqrt{c^2+a^2}}{\sqrt{b^2+c^2}+\sqrt{c^2+a^2}},0\right)$

Any plane through z-axis (i.e., $x=0,\ y=0$) is $x-ky=0$

If it passes through $F, k=\frac{a\sqrt{b^2+c^2}}{b\sqrt{c^2+a^2}}$

$\therefore$ Equation to plane OCF (the plane through z-axis and bisecting $\angle C$) is $\frac{x}{a\sqrt{b^2+c^2}}=\frac{y}{b\sqrt{c^2+a^2}}$

Similarly other two planes are $\frac{y}{b\sqrt{c^2+a^2}}=\frac{z}{c\sqrt{a^2+b^2}}$ and $\frac{z}{c\sqrt{a^2+b^2}}=\frac{x}{a\sqrt{b^2+c^2}}$

They clearly pass through the line $\frac{x}{a\sqrt{b^2+c^2}}=\frac{y}{b\sqrt{c^2+a^2}}=\frac{z}{c\sqrt{a^2+b^2}}$

Incentre of the ΔABC is the point where this line meets the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. Any point on the line is $\left(ar\sqrt{b^2+c^2},br\sqrt{c^2+a^2},cr\sqrt{a^2+b^2}\right)$. If it lies in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, $r=\frac{1}{\sum\sqrt{b^2+c^2}}$

$\therefore$ In-centre is $\left(\frac{a\sqrt{b^2+c^2}}{\sum\sqrt{b^2+c^2}},\frac{b\sqrt{c^2+a^2}}{\sum\sqrt{b^2+c^2}},\frac{c\sqrt{a^2+b^2}}{\sum\sqrt{b^2+c^2}}\right)$

Similarly, orthocenter is $\left(\frac{a^{-1}}{a^{-2}+b^{-2}+c^{-2}},\frac{b^{-1}}{a^{-2}+b^{-2}+c^{-2}},\frac{c^{-1}}{a^{-2}+b^{-2}+c^{-2}}\right)$