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IIT ACADEMY

WEEKLY ASSIGNMENT-8

SUBJECT: MATHEMATICS

TOPIC: Probability

Conceptual Problems:

1. Out of three events one and only one can happen. Given that the chance of first is $\frac{2}{3}$ of the second and the odds are 5 to 2 against the third event. Find the odds against the first event and the odds in favour of the second event.
(Ans: 5 to 2 and 3 to 4)
2. A bag contains 10 white and 3 black balls. The balls are drawn one by one without replacement till all the black balls are drawn. What is the probability that the procedure of drawing balls will come to an end at the seventh draw. Balls are considered to be distinct.
(Ans : 15/286)
3. Given two events A and B. If odds against A are as 2 : 1 and those in favour of A U B are as 3 : 1. Show that
$$\frac{5}{12} \leq P(B) \leq \frac{3}{4}$$
4. A bag contains $(2n + 1)$ coins. It is known that n of these coins have a head on both the sides whereas the remaining coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is $\frac{37}{50}$, find the value of n .
(Ans: 12)
5. From five digit numbers, a number is selected at random. What is the probability that the digits of the numbers are either in decreasing order or in increasing order?
(Ans: 189/45000)
6. A and B bet on the outcomes of the successive toss of a coin. On each toss, if the coin shows a head, A gets one rupee from B, whereas if the coin shows a tail. A pays one rupee to B. They continue to do this until one of them runs out of money. If it is assumed that the successive tosses of the coin are independent, find the probability that A ends up with all the money if A starts with five rupees and B starts with seven rupees.
(Ans: 5/12)
7. Five ordinary dice are rolled at random and the sum of the numbers shown on them is 16. What is the probability that the numbers shown on each is any one from 2, 3, 4, 5?
(Ans: 9/49)
8. A prize is to be won by A if he throws five with two dice or by B as soon as he throws 10 with 3 dice. If they throw alternately, A commencing, show that their chances of winning are equal.
9. A bag contains 10 balls of which 3 are black and 7 are white three balls are drawn from the bag one by one. Each time ball drawn is put back with 2 other balls of same colour. If the 3rd drawn ball is black, what is the probability that 1st was also black.
(Ans: 5/12)
10. A and B toss 3 coins. Find the probability that they both obtain same number of heads.
(Ans: 5/16)
11. Two events A and B are such that $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ $P(AB) = \frac{1}{4}$. Find $P(A^1/B^1)$.
(Ans: 3/4)
12. A pair of dice is thrown once each by two persons. What is the probability that sum of the numbers on the faces is same.
(Ans: 73/648)
13. A purse contains 'n' coins of unknown value; a coin drawn at random is found to be a rupee. What is the chance that it is the only rupee in the bag.
(Ans: $\frac{2}{n(n+1)}$)
14. Three identical dice are rolled. Find the probability that same number will appear on each of them.
(Ans: 3/28)
15. Ten students are taking an examination out of them 3 are excellently prepared, four are well prepared, 2 are adequately prepared and one is poorly prepared. There are 20 questions in the test paper. A student who was excellently prepared can answer all questions who was well prepared can answer 16 questions, who was adequately prepared can answer 10 questions and the one who is poorly prepared can answer only 5 questions. A student selected at random answered correctly to three randomly chosen questions. Find the probability that he was excellently prepared.
(Ans: 114/197)

16. A printing machine can print n 'Letters', $\alpha_1, \alpha_2, \dots, \alpha_n$ each letter being produced by giving a different impulse to the printer. Assume that P is the constant probability of printing the correct letter by an impulse and the impulses are independent. One of the n impulses, chosen at random, was fed into the machine twice and both times the letter α_1 was printed. Compute the probability that the impulse chosen was meant to print α_1 . ANS: $(n-1)p^2 / (np^2 - 2p + 1)$
17. A circle of radius a is inscribed in a square of side $2a$. Find the probability that a point chosen at random is inside the square but outside the circle. (Ans: $\frac{4 - \pi}{4}$)
18. Let A be a set with n elements and let 3 subsets P, Q, R of A are chosen at random. What is the probability that $P \cup Q \cup R = A$. (Ans: $(7/8)^n$)
19. Ten percent of patients referred to a pathology clinic with suspected Krupp's disease are actually suffering from it. Of these six percent have the more severe lymphatic form of the disease and the other 4% have the non-lymphatic form. A new test is available which gives a positive result with probability one for a patient with lymphatic Krupp's with probability one for a patient with lymphatic Krupp's with probability $3/4$ for a patient with non lymphatic Krupp's and with prob. $1/2$ for a patient without the disease. Given that the test gave a positive result for a certain patient what is the probability that he has Krupp's disease. (Ans: $1/6$)
20. A die is rolled then n coins are tossed where n is the number shown on the die. What is the probability of getting exactly 2 heads. (Ans: $33/128$)
21. A group of 25 athletes represent a country in the Olympic games, 5 of them takes steroids. In a track and field event athlete wins a gold medal. The remaining 24 are taken for a test to detect the intake of steroids 2 persons each undergo the test at a time. The first two persons selected at random from the 24 are found positive to this test. What is the probability that the gold medalist took the steroids. (Ans: $3/23$)
22. 10% of valves produced by a factory A are defective and 20% of valves produced by factory B are defective. A bag contains 4 valves of fact. A and 5 of factory B. If two valves are drawn from the bag at random find the probability that at least one is defective. (Ans: $517/1800$)
23. n positive integers are taken at random and multiplied together. Show that the chance of the last digit in the product being 0 is $(2^n - 1)(5^n - 4^n)/10^n$.
24. If A and B are any two events then show that

$$\begin{aligned} P(A \cap B) - P(A)P(B) &= P(A)P(B^c) - P(A \cap B^c) \\ &= P(A^c)P(B) - P(A^c \cap B) \\ &= P(A \cup B)^c - P(A^c)P(B^c) \end{aligned}$$
25. Set A contains all the 4 digit natural numbers and set B contains the 4 digit natural numbers, each of which is a square of a natural number. A number chosen at random from one of these sets was found to have its first two digits identical of one kind and the next two digits identical of another kind. What is the probability that the chosen number was from set A . (Ans: $17/42$)
26. An urn contains 10 white, 9 black, 8 red and 3 blue balls, balls are drawn one by one at random from the urn until 2 blue balls are obtained. Find the probability of drawing 2 blue balls at 11th draw. (Ans: $\frac{3 \times {}^{27}C_9}{10 \times {}^{30}C_{10}}$)
27. If A and B are disjoint events, and $P(A \cup B) \neq 0$, show that $P(A/A \cup B) = P(A)/[P(A) + P(B)]$.
28. A and B throw with a pair of dice. If A wins by throwing 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is $30/61$.
29. Among two room mates A and B , A always takes his umbrella when he goes out while B forgets his umbrella with probability $1/2$. After visiting 3 shops they returned home. What is the probability that
 i) both of them have their umbrellas
 ii) B has lost his umbrella given that they returned with one umbrella. (Ans: $\frac{2457}{8192}, \frac{999}{7218}$)

30. Two hunters A and B shot a bear simultaneously. The bear was shot dead with only one hole in its hide. The probability of A shooting the bear is 0.8 and that of B shooting it is 0.4. The hide was sold for Rs 600 and this amount was divided between A and B fairly. If A's share is Rs P, find $[P]$ where $[]$ denotes step function. Ans: 514
31. A box contains a fair coin and a two headed coin. A coin is selected at random and tossed. If head appears, the other coin is tossed. If tail appears, the same coin is tossed. (i) Find the probability that head appears on second toss. (ii) If head appeared on second toss, find the probability that it also appeared on the first toss.
32. A plane is landing. If the weather is fair, the pilot can see the runway and the probability of safe landing in this case is $\frac{3}{4}$. If there is a low cloud ceiling the pilot has to make a blind landing by instruments. The probability of failure free functioning of instruments is $\frac{4}{5}$. If the instruments function normally the probability of blind landing is same as that of the visual landing (which is $\frac{3}{4}$) otherwise the probability of safe landing is $\frac{2}{3}$. Find the probability of safe landing, if it is known that the probability of low cloud ceiling is $\frac{1}{4}$. Ans: $\frac{179}{240}$
33. The probabilities of two students A and B solving a problem are $\frac{1}{6}$ and $\frac{1}{8}$ respectively. If the probability of their making a common error is $\frac{1}{525}$ and if they obtain the same answer, find the probability that their answer is correct. Ans: $\frac{15}{16}$
34. A man wants to marry a girl with two independent qualities: fair complexion and handsome dowry. The probability of getting a girl only with the first quality is $\frac{1}{12}$ and probability of getting a girl only with the second quality is $\frac{1}{2}$. Find the probability of his getting married to a girl with both qualities. Ans: $\frac{1}{6}$ or $\frac{1}{4}$
35. A player tosses a coin and is to score one point for every head turned up and two for every tail. He is to play on until his score reaches or passes n. If P_n is the chance for containing exactly n, how that $P_n = \frac{1}{2}(P_{n-1} + P_{n-2})$ and hence find the value of P_n . Also find the limiting value as $n \rightarrow \infty$.
(Ans: $P_n = \frac{1}{3}\{2 + (-1)^n / 2^n\}$ and $\frac{2}{3}$)
36. A box contains two good and two bad lamps. A lamp is drawn at random from the box, if it is good it is kept back in the box, other wise it is replaced by a good lamp taken from another box. What is the expected number of good lamps in the box after three trails? (Ans: $\frac{101}{32}$)
37. Suppose that a die is rolled twice. Let X be the random variable defining the maximum value obtained in the two throws. Find the mean and variance of X. (Ans: $\frac{161}{36}$)
38. Two points are taken at random on the circumference of a circle. Find the probability that their distance apart is greater than the radius of the circle. (Ans: $\frac{2}{3}$)

(Hints/Solutions will be put up on 29-02-2012)