KRISHNA MURTHY IIT ACADEMY

MODEL TEST FOR IIT(JEE)-2010

UNIT – 4 :: PAPER – II

27th JANUARY, 2010

Answers and Solutions :
Chemistry:

1. Ans. : D

2. Ans. : A

3. Ans. : C

4. Ans. : C

5. Ans. : ABC

6. Ans. : A

7. Ans. : B

8. Ans. : BC

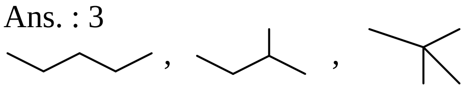
9. Ans. : AC
Sol. This reaction is given by esters

10. Ans. : $A \rightarrow P,S,\ B \rightarrow R,\ C \rightarrow Q,\ D \rightarrow S$

11. Ans. : $A \rightarrow Q,R,\ B \rightarrow R,\ C \rightarrow S,\ D \rightarrow P$

12. Ans. : 4

13. Ans. : 2

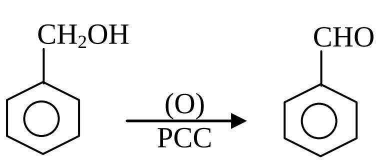
14. Ans. : 3
Sol. 

15. Ans. : 5
Sol. Let % of 2R chlorobutane = x
$x \times 30 + (1 - x) \times (-30) = -27.$
$x = 1/20$
% of 2R chlorobutane = 5%.

16. Ans. : 3

17. Ans. : 4

18. Ans. : 5
Sol. $CH_3CHO + HCHO(excess) \rightarrow (HOCH_2)_3C - COO^- + (HO(H_2)_3\text{–}C\text{–}CH_2OH.$

19. Ans. : 2
Sol. 

MAT. : : 20) A, 21) D, 22) D, 23) A, 24) AB, 25) BC, 26) ABD, 27) AD, 28) BC, 29) A – Q, B – P, C – S, D – R; 30) A – Q, B – R, C – P, D – S; 31) 0, 32) 8, 33) 3, 34) 1, 35) 2, 36) 4, 37) 3, 38) 0.

SOLUTIONS :
.
20. Ans. : A

Sol. $\sin^{-1}\frac{1+x^2}{2x}$ is defined for $\left|\frac{1+x^2}{2x}\right| \le 1$

$\rightarrow$ x = ± 1.

Out of these two values of x, only x = 1 satisfies the given equation.

21. Ans. : D

Sol. $(r_1 + r_2) = 4R\left(\sin\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}+\sin\frac{B}{2}.\cos\frac{A}{2}.\cos\frac{C}{2}\right)$

$= 4R\left[\cos\frac{C}{2}\left(\sin\frac{A}{2}.\cos\frac{B}{2}+\sin\frac{B}{2}.\cos\frac{A}{2}\right)\right]$

$= 4R\cos\frac{C}{2}\sin\left(\frac{A+B}{2}\right) = 4R\cos^2\frac{C}{2}$

Similarly,

$(r_2 + r_3) = 4R\cos^2\frac{A}{2}$, $(r_3 + r_1) = 4R\cos^2\frac{B}{2}$.

Also, a + b + c = 2R(sin A + sin B + sin C)

$= 16R\cos\frac{A}{2}.\cos\frac{B}{2}.\cos\frac{C}{2}$.

$\rightarrow 4S^2 = 256R^2 .\ \cos^2\frac{A}{2}.\cos^2\frac{B}{2}.\cos^2\frac{C}{2}$

$\rightarrow RS^2 = 64R^3 .\ \cos^2\frac{A}{2}.\cos^2\frac{B}{2}.\cos^2\frac{C}{2}$

$\rightarrow \frac{11(r_1+r_2)}{RS^2} = 1$

22. Ans. : D

Sol. AD = C . Sin B.

$\angle$DPE = π – A (as $\angle$ADP = $\angle$AEP = $\frac{\pi}{2}$)

Using sine rule in triangle DPE, we get

$\frac{DE}{\sin(\pi - A)} = 2R' = AD = C . \text{Sin B}.$

$\rightarrow$ DE = C sin B . sin A = C . $\frac{b}{2R}$. Sin A.

$= \frac{\Delta}{R}$.

23. Ans. : A

Sol. From the figure

$\frac{z_1 - 0}{\bar{z}_1 - 0} = \frac{z_1 - 0}{\bar{z}_1 - 0}e^{2i\,\pi/n} = e^{2\pi\, i/n} \rightarrow \frac{z_1}{\bar{z}_1} = e^{2\pi/n}$ ……. (1)

Let $z_1 = re^{i\theta}$, then

$\therefore \frac{z_1}{\bar{z}_1} = \frac{re^{i\theta}}{re^{-i\theta}} = e^{2i\theta} = e^{2\pi i/n}$

$\therefore \theta\ \frac{\pi}{n}$

$\rightarrow \frac{IM(z_1)}{\text{Re}(z_2)} = \sqrt{2} - 1 \quad \rightarrow \frac{r\sin(\pi/n)}{r\cos(\pi/n)} = \sqrt{2} - 1 \rightarrow \tan\frac{\pi}{n} = \tan\frac{\pi}{8}$

$\therefore$ n = 8.

24. Ans. : AB

Sol. $\because |z_1| = |z_2| = |z_3| \rightarrow$ origin is the circumcircle of ΔABC

$\rightarrow$ Orthocentre of ΔABC is $z_1 + z_2 + z_3$

Now $\angle$AHB = $\angle$CHA

$\rightarrow \arg\left(\frac{z_1+z_2+z_3-z_2}{z_1+z_2+z_3-z_1}\right) = \arg\left(\frac{z_1+z_2+z_3-z_1}{z_1+z_2+z_3-z_3}\right)$

$\rightarrow \arg\left(\frac{(z_1+z_3)(z_1+z_2)}{(z_2+z_3)^2}\right) = 0.$

Similarly $\angle AOB = \angle COA \rightarrow \frac{z_2 z_3}{z_1^2}$ is purely real.

25. Ans. : BC

Sol. Let $a = a_1 + ia_2$ and $c = c_1 + ic_2$, then

Slope of the line $a\bar{z} + \bar{a}z + b = 0$ is $\frac{a_1}{a_2}$ and slope of the line $c\bar{z} + \bar{c}z + d = 0 = \frac{c_1}{c_2}$.

So, $-\frac{a_1}{a_2}\times\frac{c_1}{c_2} = -1 \rightarrow a_1c_1 + a_2c_2 = 0$

$\rightarrow \left(\frac{a+\bar{a}}{2}\right)\left(\frac{c+\bar{c}}{2}\right) + \left(\frac{a-\bar{a}}{2i}\right)\left(\frac{c-\bar{c}}{2i}\right) = 0 \rightarrow a\bar{c} + \bar{a}c = 0$

$\therefore a\bar{c}$ is purely imaginary. Also $\frac{a}{c} = -\frac{\bar{a}}{\bar{c}}$

$\rightarrow \frac{a}{c}$ is also purely imaginary $\rightarrow \arg\left(\frac{a}{c}\right) = \pm\frac{\pi}{2}$.

26. Ans. ABD

Sol. Let $z = x + iy$; $\arg(z) = \frac{\pi}{6} \rightarrow \frac{y}{x} = \tan\frac{\pi}{6} \rightarrow y = \frac{1}{\sqrt{3}}x$, which is a straight line. Also, $\left|z - 2\sqrt{3}i\right| = r$, represents a circle with centre at $(0, 2\sqrt{3})$ and radius r. The straight line will intersect the circle if the perpendicular distance from the centre on the line < r.

$\rightarrow \left|\frac{0-2\sqrt{3}.\sqrt{3}}{2}\right| < r \quad \rightarrow r > 3$. Therefore $[r] \geq 3$.

27. Ans. : AD

Sol. Given, $a^4 + b^4 + c^4 = 2a^2(b^2 + c^2)$

$\rightarrow a^4 + b^4 + c^4 - 2a^2c^2 + 2b^2c^2 = 2b^2c^2$

$\rightarrow (b^2 + c^2 - a^2)^2 = 2b^2c^2$

$\rightarrow b^2 + c^2 - a^2 = \sqrt{2}\,bc$ or $b^2 + c^2 - a^2 = -\sqrt{2}\,bc$

$\rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{1}{\sqrt{2}}$ or $\rightarrow \frac{b^2+c^2-a^2}{2bc} = -\frac{1}{\sqrt{2}}$

$\rightarrow \cos A = \frac{1}{\sqrt{2}} = \cos 45^o$ or $\cos A = -\frac{1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

$A = 45^o$ or $A = 135^o$.

28. Ans. : BC

Sol. Since, $\cos\theta\ \frac{2\cos(\theta-\alpha)\cos(\theta+\alpha)}{\cos(\theta-\alpha)+\cos(\theta+\alpha)} = \frac{2\cos^2\theta-\sin^2\alpha}{2\cos\theta\cos\alpha}$

$\rightarrow \cos^2\theta\cos\alpha = \cos^2\theta - \sin^2\alpha$

$\rightarrow \cos^2\theta = \frac{\sin^2\alpha}{1-\cos\alpha}$

$\rightarrow \cos^2\theta = \frac{4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}$

$\rightarrow \cos^2\theta\sec^2\frac{\alpha}{2} = 2$

$\rightarrow \cos\theta\sec\frac{\alpha}{2} = \pm\sqrt{2}$.

29. Ans. : A – Q, B – P, C – S, D – R

Sol. Since, $(G_3A)^2 = (G_3M)(G_3C)$

$\therefore \quad \left(\frac{c}{2}\right)^2 = \frac{1}{3}(G_3C)^2$

$\therefore \frac{c^2}{4} = \frac{1}{3}\left(\frac{2b^2+2a^2-c^2}{4}\right) \rightarrow a^2 + b^2 = 2c^2 \qquad \text{...... (i)}$

Now, in ΔAG_1C

$$\sin \angle CBM = \frac{a \sin C}{2(AG_1)}$$

and in Δ BCG_2

$$\sin \angle CBM = \frac{a \sin C}{2(AG_1)}$$

and in Δ BCG_2

$$\sin \angle CBM = \frac{b \sin C}{2(BG_2)}$$

Also, $AG_1 = \sqrt{\frac{2b^2 + 2c^2 - a^2}{4}} = \frac{\sqrt{3}}{2} b$ (using eq. (i))

and $BG_2 = \sqrt{\frac{2b^2 + 2c^2 - b^2}{4}} = \frac{\sqrt{3}}{2} a$

Now, in $\angle CBM + \sin \angle CBM = \frac{2\Delta}{\sqrt{3}} \left(\frac{a^2 + b^2}{a^2 b^2} \right)$

$$= \frac{1}{\sqrt{3}} \left(\frac{a^2 + b^2}{ab} \right) \sin C \quad \text{.......... (ii)}$$

Also, $\cos C = \frac{a^2 + b^2 - c^2}{2ab} \rightarrow a^2+b^2 - 4ab \cos C.$

So, from Eq(ii)

$$\text{Sin } \angle CAM + \sin \angle CBM = \frac{2}{\sqrt{3}} \sin 2C \le \frac{2}{\sqrt{3}} \quad \left(\text{where } C = \frac{\pi}{4} \right)$$

30. Ans. : A – Q, B – R, C – P, D – S

Sol. A. $(z + \alpha\beta)^3 = \alpha^3 \rightarrow z + \alpha\beta = \alpha, \omega\alpha, \omega^2\alpha$

$\rightarrow z = \alpha - \alpha\beta, \omega\alpha - \alpha\beta, \omega^2\alpha - \alpha\beta$, say z_1, z_2, z_3 respectively.

Now, $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1| = \sqrt{3}|\alpha|$.

So, the triangle is equilateral and has area = $\frac{\sqrt{3}}{4} |z_1 - z_2|^2$.

B. $|z - \alpha|^2 = 4|z - \bar{\alpha}|^2 \rightarrow z\bar{z} - \alpha\bar{z} + \alpha\bar{\alpha}$

$= 4(z\bar{z} - \alpha z - \overline{\alpha z} + \alpha\bar{\alpha})$

$\rightarrow 3z\bar{z} + (\alpha - 4\bar{\alpha})\bar{z} + (\bar{\alpha} - 4\alpha)z + 3\alpha\bar{\alpha} = 0$

or $z\bar{z} + \frac{\alpha - 4\bar{\alpha}}{3}\bar{z} + \frac{\bar{\alpha} - 4\alpha}{3} z + \alpha\bar{\alpha} = 0$

Which is a circle of radius

$$= \sqrt{\left|\frac{\alpha - 4\bar{\alpha}}{3}\right|^2 - \alpha\bar{\alpha}} = \sqrt{-\frac{4}{9}(\alpha - \bar{\alpha})^2} = \frac{2}{3}|\alpha - \bar{\alpha}|.$$

C. z lies on a circle of radius 1 and centre at (1, 0)

$\angle OPA = \pm \frac{\pi}{2} \rightarrow \frac{2 - z}{0 - z} = \frac{|2 - z|}{|z|} e^{\pm i \frac{\pi}{2}}$

$\rightarrow \frac{z - 2}{z} = \frac{AP}{OP}(\pm i) = \pm 1 \tan \alpha$

$\therefore \left| \frac{z - 2}{z} \right| = |\tan \alpha|$

D. $z_1 + z_2 = -p$ and $z_1 z_2 = q$

Also, $\frac{z_2}{z_1} = \cos \alpha \pm i \sin \alpha \rightarrow \frac{z_2 - z_1 \cos \alpha}{z_1} = \pm i \sin \alpha$

or $z_2^2 - 2z_2 z_1 \cos \alpha + z_1^2 \cos^2 \alpha = -z_1^2 \sin^2 \alpha$

$\rightarrow z_1^2 + z_2^2 = 2z_1 z_2 \cos \alpha$

or $(z_1 + z_2)^2 = 2z_1 z_2(1 + \cos \alpha) \rightarrow \frac{p^2}{p} = 4\cos^2 \frac{\alpha}{2}$.

31. Ans. : 0

Sol. Since, $u = \frac{\pi}{2} - \tan^{-1} \left\lfloor \sqrt{\cos \alpha} \right\rfloor - \tan^{-1} \left(\sqrt{\cos \alpha} \right)$

$\rightarrow 2\tan^{-1}\left(\sqrt{\cos\alpha}\right) = \dfrac{\pi}{2} - u$

Now, $\cos\left(\dfrac{\pi}{2} - u\right)$ - $\cos\left(2\tan^{-1}\sqrt{\cos\alpha}\right)$

$\rightarrow \sin u = \dfrac{1-\tan^2\left(\tan^{-1}\sqrt{\cos\alpha}\right)}{1+\tan^2\left(\tan^{-1}\sqrt{\cos\alpha}\right)}$

$= \dfrac{1-\left(\sqrt{\cos\alpha}\right)^2}{1+\left(\sqrt{\cos\alpha}\right)^2} = \dfrac{1-\cos\alpha}{1+\cos\alpha} = \tan^2\dfrac{\alpha}{2}$

$\rightarrow \sin u - \tan^2\dfrac{\alpha}{2} = 0.$

32. Ans. : 8

Sol. Since, $(\sin^{-1}x)^3 + (\cos^{-1}x)^3 = a\pi^3$.

$(\sin^{-1}x + \cos^{-1}x)^3 - 3\sin^{-1}x\cos^{-1}x(\sin^{-1}x + \cos^{-1}x) = a\pi^3$

$\rightarrow \dfrac{\pi^2}{4} - 3\sin^{-1}x \,.\, \cos^{-1}x = 2a\pi^2$

$\rightarrow \sin^{-1}x\left(\dfrac{\pi}{2} - \sin^{-1}x\right) = \dfrac{\pi^2}{12}(1-8a)$

$\rightarrow (\sin^{-1}x)^2 - \dfrac{\pi}{2}\sin^{-1}x = -\dfrac{\pi^2}{12}(-8a)$

$\rightarrow \left(\sin^{-1}x - \dfrac{\pi}{4}\right)^2 = \dfrac{\pi^2}{12}(8a-1) + \dfrac{\pi^2}{16}$

$\rightarrow \left(\sin^{-1}x - \dfrac{\pi}{4}\right)^2 = \dfrac{\pi^2}{48}(32a-4+3)$

$\rightarrow \left(\sin^{-1}x - \dfrac{\pi}{4}\right)^2 = \dfrac{\pi^2}{48}(32a-1)$

As, $\sin^{-1}x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\rightarrow -\dfrac{3\pi}{4} \le \sin^{-1} - \dfrac{\pi}{4} \le \dfrac{\pi}{4}$

$\rightarrow 0 \le \left(\sin^{-1}x - \dfrac{\pi}{4}\right)^2 \le \dfrac{9\pi^2}{16}$

$\rightarrow 0 \le (32a - 1) \le 27$

$\rightarrow \dfrac{1}{32} \le a \le \dfrac{7}{8}$

Thus, the required set of values of a is $\left[\dfrac{1}{32}, \dfrac{7}{8}\right]$

On comparing with $\left[\dfrac{\lambda}{32}, \dfrac{\mu}{8}\right]$, we get

$\lambda = 1$ and $\mu = 7$

$\therefore \quad \lambda + \mu = 8$

33. Ans. : 3

Sol. We have $\dfrac{\pi}{4} = \arg.\dfrac{z-z_1}{z-z_2} = \arg.(z - z_1) - \arg(z - z_2)$

$\rightarrow \tan^{-1} 1 = \tan^{-1}\dfrac{y-6}{x-10} - \tan^{-1}\dfrac{y-6}{x-4}$ - $\tan^{-1}\left[\dfrac{\dfrac{y-6}{x-10} - \dfrac{y-6}{x-4}}{1+\dfrac{(y-6)}{(x-10)}\dfrac{(y-6)}{(x-4)}}\right]$

$\rightarrow 1 = \dfrac{6(y-6)}{x^2+y^2-14x-12y+76}$

$\rightarrow x^2 + y^2 - 14x - 18y + 112 = 0,$

or $(x - 7)^2 + (y - 9)^2 = 18 = \left(3\sqrt{2}\right)^2$

or $|x - 7 + i(y - 9)| = 3\sqrt{2}$

$\rightarrow |z - 7 - 9i| = 3\sqrt{2}$.

34. Ans. : 1

Sol. L.H.S. = $iz^2(z - i) - (z - i) = (z - i)(iz^2 - i) = 0$

$\rightarrow$ Either $z - i = 0 \rightarrow z = i \rightarrow |z| = 1$

or $iz^2 - 1 = 0 \rightarrow iz^2 = 1 \rightarrow iz^2(-i\,\bar{z}^2) = 1$

$\rightarrow (z\bar{z})^2 = 1 \rightarrow |z| = 1.$

35. Ans. : 2

Sol. LHS = $\dfrac{\sin\theta}{\cos 3\theta} + \dfrac{\sin 3\theta}{\cos 9\theta} + \dfrac{\sin 9\theta}{\cos 27\theta}$

Now, $\dfrac{\sin\theta}{\cos 3\theta} = \dfrac{\sin\theta}{\cos 3\theta} \times \dfrac{2\cos\theta}{2\cos\theta}$

$= \dfrac{\sin 2\theta}{2\cos 3\theta\cos\theta} = \dfrac{1}{2}\dfrac{\sin(3\theta - \theta)}{\cos 3\theta\cos\theta}$

$= \dfrac{1}{2}\left[\dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\cos 3\theta\cos\theta}\right]$

$= \dfrac{1}{2}(\tan 3\theta - \tan\theta)$

Similarly, $\dfrac{\sin 3\theta}{\cos 9\theta} = \dfrac{1}{2}(\tan 9\theta - \tan 3\theta)$

and $\dfrac{\sin 9\theta}{\cos 27\theta} - \dfrac{1}{2}(\tan 27\theta - \tan 9\theta)$

$\therefore \dfrac{\sin\theta}{\cos 3\theta} + \dfrac{\sin 3\theta}{\cos 9\theta} + \dfrac{\sin 9\theta}{\cos 27\theta}$

$= \dfrac{1}{2}(\tan 27\theta - \tan 9\theta)$

= RHS $\quad\therefore k = 2.$

36. Ans. : 4

Sol. Let $\theta - \dfrac{\pi}{6} \rightarrow 8\theta = \dfrac{\pi}{2},$

$\therefore \tan^2\dfrac{\pi}{16} = \tan^2\theta$

and $\tan^2\dfrac{7\pi}{16} = \tan^2\left(\dfrac{\pi}{2} - \dfrac{\pi}{16}\right) = \tan^2\left(\dfrac{\pi}{2} - \theta\right) = \cot^2\theta$

$\therefore \tan^2\dfrac{\pi}{16} + \tan^2\dfrac{7\pi}{16}$

$= \tan^2\theta + \cot^2\theta = \dfrac{8}{1 - \cos 4\theta} - 2.$

$\therefore \tan^2\dfrac{\pi}{16} + \tan^2\dfrac{7\pi}{16} = \dfrac{8}{1 - \cos 4\left(\dfrac{\pi}{16}\right)} - 2 = 14 + 8\sqrt{2}$

Applying similar process with other terms, we get $\tan^2\dfrac{2\pi}{16} + \tan^2\dfrac{6\pi}{16} = 6$

and $\tan^2\dfrac{3\pi}{16} + \tan^2\dfrac{5\pi}{16} = 14 - 8\sqrt{2}$.

Adding, we get 34. 34 – 30 = 4.

37. Ans. : 3

Sol. Given $\sin x\cos 3x - \alpha\cos x\sin 3x = 0$ …… (i)

$\rightarrow \alpha = \dfrac{\sin x\cos 3x}{\cos x\sin 3x} = \dfrac{\tan x}{\tan 3x} = \dfrac{\tan x(1 - 3\tan x)}{3\tan x - \tan^3 x}$

$\alpha = \dfrac{1 - 3\tan^2 x}{3 - \tan^2 x}$ …… (ii)

For real value of x, RHS of equation (ii) never lies between $\left(\dfrac{1}{3}, 3\right)$

Hence equation (i) is not solvable for any real value of x if $\in \left(\dfrac{1}{3}, 3\right)$.

38. Ans. : 0

Given sin x + 2 sin 2x = 3 + sin 3x
$\rightarrow$ sin 3x – sin x – 2sin 2x + 3 = 0
$\rightarrow$ 2sin x . cos 2x – 4 sin x . cos x + 3 = 0
$\rightarrow$ sin x (2 cos 2x – 4 cos x) + 3 = 0
$\rightarrow$ sin x {2(2 $\cos^2$ x – 1) – 4 cos x} + 3 = 0
$\rightarrow$ sin x (4 $\cos^2$ x – 4 cos x – 2) + 3 = 0
$\rightarrow$ sin x $(2 \cos x - 1)^2$ + 3(1 – sin x) = 0 ……. (ii)

Since $0 \le x \le \pi$
$\rightarrow$ $0 \le \sin x \le 1$
$\therefore$ $1 - \sin x > 0$
Also, $(2 \cos x - 1)^2 \ge 0$

Hence, equation (i) holds true only if
Sin x $(2 \cos x - 1)^2 = 0$ ….. (ii)
and 3(1 – sin x) = 0 ……(iii)

From equation (iii)
sin x = 1
$\rightarrow$ cos x = 0
sin x(2cos x – $1)^2$) = 1 $\neq$ 0

Thus, equation (ii) and (iii) can never be satisfied simultaneously.
Hence, no solution is possible.

PHY. : 39) D; 40) D, 41) C; 42) A; 43) ABCD; 44) ACD; 45) ABD; 46) ABCD; 47) ABCD; 48) A – PR, B – Q, C – QS, D – Q; 49) A – P, B – PR, C – QR, D – S; 50) 1; 51) 5; 52) 4; 53) 1; 54) 2; 55) 6; 56) 1; 57) 6.

ANSWERS AND SOLUTIONS :

39. Ans. : D
Sol. $V - iR - iR_0 - V_0 = 0$ $\qquad i = \dfrac{V - V_0}{R + R_0}$

$V - iR - \dfrac{q}{C} - V = 0$ $\qquad q = -iRC.$

40. Ans. : D
Sol. A_1 shorts the terminals as it is ideal.
$\therefore$ Current in 10Ω = 0 $\qquad \therefore$ Reading = 0

41. Ans. : C
Sol. Magnetic dipole moment

$$\bar{\mu} = i_0 l^2(-\hat{k}) + i_0 \frac{l^2}{2}(-\hat{j}) + i_0 \frac{l^2}{2}(-\hat{i})$$

$$= i_0 l^2\left(-\frac{\hat{i}}{2} - \frac{\hat{j}}{2} - \hat{k}\right)$$

$$\bar{\tau} = \bar{\mu} \times B_0 \hat{i} = i_0 B_0 l^2\left(-\hat{i} + \frac{\hat{k}}{2}\right)$$

42. Ans. : A
Sol. $\oint_C \bar{B}.\bar{dl} = \mu_0[+1 + 5 - 2] = 4\mu_0.$

43. Ans. : ABCD
Sol. $Z = \sqrt{R^2 + \left(R - \dfrac{R}{3}\right)^2} = \dfrac{\sqrt{13}}{3} R$ $\quad \cos\phi = \dfrac{3}{\sqrt{13}}$

Power supplied = $\dfrac{V_0 I_0 \cos\phi}{2}$

$I_0 = \dfrac{V_0}{Z} = \dfrac{3V_0}{\sqrt{13}R}$ $\qquad$ Power = $\dfrac{9V_0^{\,2}}{26R}$

$$V_{ab,0} = I_0(X_L - X_C)$$

$$= \frac{3V_0}{\sqrt{13}R}\left(R - \frac{R}{3}\right) = \frac{2V_0}{\sqrt{13}}$$

$$v_{ab} = \frac{2V_0}{\sqrt{13}}\sin\left(\omega t + \frac{\pi}{2} - \phi\right) = \frac{2V_0}{\sqrt{13}}\cos(\omega t - \phi)$$

44. Ans. : ACD

Sol. At any time t $i > 0$

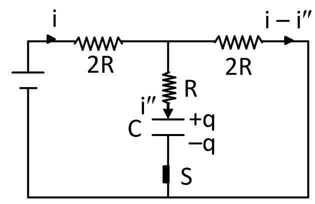
Time period = $\frac{2\pi}{(\pi/T)} = 2T$

$$i_{av} = \frac{\int_0^{2T} i\,dt}{2T} \neq 0. \qquad i_{rms} = \sqrt{\frac{\int_0^{T} i^2 dt}{T}}$$

$$= i_0\sqrt{\frac{3}{2}+\frac{4}{\pi}}$$

45. Ans. : ABD

Sol. At t = 0 when S is closed



$V_0 - 2i_0R - i'R = 0$

$V_0 - 2i_0'R - (i_0' - i')(2R) = 0$

$\therefore i_0' = \frac{3V_0}{8R}$

At time t

$V_0 - 2iR - i''R - \frac{q}{C} = 0.$

$V_0 - 2iR - (i - i'')(2R) = 0$

$\therefore 4i''R + \frac{2q}{C} = V_0$

$i'' = \frac{dq}{dt} \qquad \therefore \frac{dq}{dt} = \frac{V_0}{4R} - \frac{q}{2RC}$

Steady state after S is closed

$V_0 - 2i_S'R - 2i_S'R = 0 \qquad i_S' = \frac{V_0}{4R}$

$V_0 - 2i_S'R - \frac{q_S}{C} = 0 \qquad q_S = \frac{CV_0}{2}.$

46. Ans. : ABCD

Sol. $B(2\pi r) = \mu_0 i' \qquad 0 \le r \le R$

$i' = \int_0^r \sigma(2\pi r' dr')$

$B = \mu_0\sigma_0\left(\frac{r}{2} - \frac{r^3}{4R^2}\right)$

$\frac{dB}{dr} = 0 \qquad$ for $r = \sqrt{\frac{2}{3}}\,R < R$

$\left.\frac{dB}{dr}\right)_{r=0} = \frac{\mu_0\sigma_0}{2} \qquad \left.\frac{dB}{dr}\right)_{r=R} = -\frac{\mu_0\sigma_0}{4}$

$r \ge R \qquad B(2\pi r) = \mu_0 i'_0 \qquad B = \frac{\mu_0 i'_0}{2\pi r}$

$\therefore$ The graph is correct

$$i'_0 = \int_0^R \sigma(2\pi r)dr = \frac{\pi\sigma_0 R^2}{2}$$

47. Ans. : ABCD

Sol. $\overline{F}_m = 0$ in a uniform $\overline{B}$.

$\overline{\tau} = \overline{\mu} \times \overline{B} = \frac{i'_0 B_0 ab}{2}\left(\sqrt{3}\hat{i} - \hat{j}\right)$

Force on PS $\qquad \overline{F}_{PS} = i'_0\left(-a\hat{j}\right) \times \overline{B}$

$$= -\frac{i'_0 B_0 a\hat{k}}{2}$$

Potential energy $U_m = -\overline{\mu}.\overline{B} = 0.$

48. Ans. : A – PR, B – Q, C – QS, D – Q

Sol. U_i for $C_1 = \frac{1}{2}C_1\left(\frac{2V}{3}\right)^2 = \frac{4V^2}{9}$

U_i for $C_2 = \frac{2V^2}{9}$

When separation is doubled capacitance is halved.

$U_{f_1} = \frac{8V^2}{25}$ $\quad U_{f_2} = \frac{2V^2}{25}$.

49. Ans. : A – P, B – PR, C – QR, D – S

50. Ans. : 1

Sol. Flux of $\overline{E}$ over pair of faces ⊥ x-axis

$= 2\alpha a^3 - \alpha a^3 = \alpha a^3$

Similarly the y and z faces. βa^3 and γa^3

$\therefore \phi = (\alpha + \beta + \gamma)a^3$

51. Ans. : 5

Sol. $q_H = \frac{2Q + 3Q - Q + 6Q}{2} = 5Q.$

52. Ans. : 4

Sol. $E(4\pi r^2) = \frac{q}{\in}$

$$q = \int_0^r \left(4\pi x^2 dx\right)\rho = \int_0^r \rho_0 4\pi\left(x - \frac{x^2}{R}\right)x^2 dx$$

$$= 4\pi\left[\frac{x^4}{4} - \frac{x^5}{5R}\right]_0^r$$

$$= 4\pi\rho_0\left[\frac{r^4}{4} - \frac{r^5}{5R}\right]$$

$$E = \frac{\rho_0}{\in}\left[\frac{r^4}{4} - \frac{r^5}{5R}\right].$$

53. Ans. : 1

Sol. $q_1 + q_2 = Q$

$$U = \frac{q_1^2}{8\pi\varepsilon_0 R_1} + \frac{q_2^2}{8\pi\varepsilon_0 R_2} + \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

For max/min $\frac{dV}{dq_1} = 0$

$q_1 \approx \frac{qR_2}{R_1 + R_2}$, $\quad q_2 \approx \frac{qR_1}{R_1 + R_2}$ (r is large)

$\frac{q_1}{q_2} = \frac{R_1}{R_2}$.

54. Ans. : 2

Sol. Charge flown through $B_1 = 2V_0$ μC

Charge flown through $B_2 = 0$

$W_{B_1} = -(2V_0)(V_0)$

$= -2V_0^2$ μJ.

55. Ans. : 6

Sol. $U_L = \frac{1}{2} L i^2$

$\frac{dV_L}{dt} = \frac{1}{2} L\, 2i \frac{dt'}{dt}$

$= 6 \times 10^{-3}$ (5) (200)

= 6 watts.

56. Ans. : 1

Sol. $i = \frac{V_0}{R}\left(1 - e^{\frac{Rt}{L}}\right)$

$$\left(1-\frac{1}{e}\right)\frac{V_0}{R} = \frac{V_0}{R}\left(1-e^{\frac{Rt}{2}}\right)$$

$$t = \frac{L}{R}.$$

57. Ans. : 6

Sol. Steady state = + 8 – 3 – $\frac{q}{C}$ = 0

Q = 5(6) μC.