KRISHNA MURTHY IIT ACADEMY

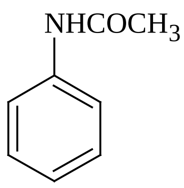
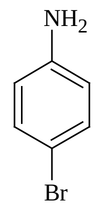
25-01-2010

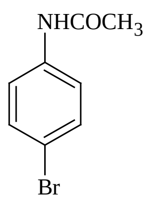
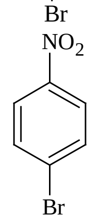
- CHEMISTRY : ANSWER AND SOLUTIONS :

1. Ans. : B

2. Ans. : D

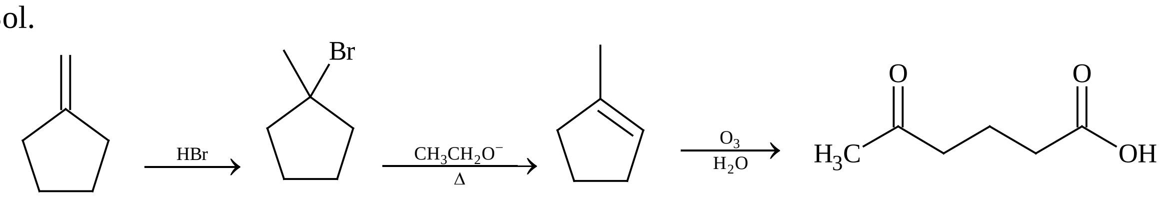
Sol. the compounds are :

. A  C 

B  D 

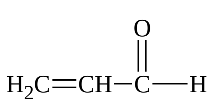
3. Ans. : B

Sol.

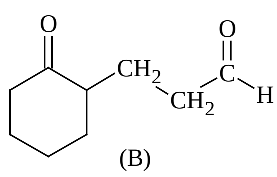


4. Ans. : A

Sol.



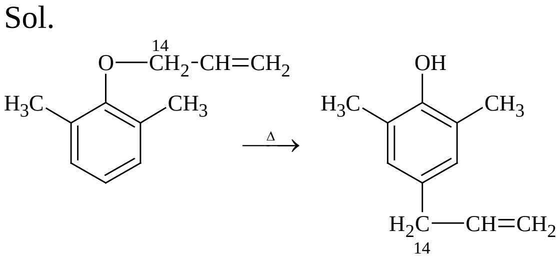
(A)



(B)

5. Ans. : A

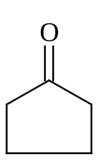
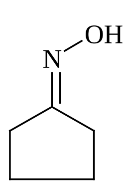
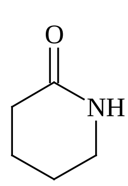
Sol.



(After rearrangement)

6. Ans. : A

Sol.

(A)  (B)  (C) 

7. Ans. : B

8. Ans. : D

9. Ans. : ABD

10. Ans. : AB

11. Ans. : ABCD

12. Ans. : ABD

13 Ans. : A

14. Ans. : B

15. Ans. : D

16. Ans. : B

17. Ans. : B

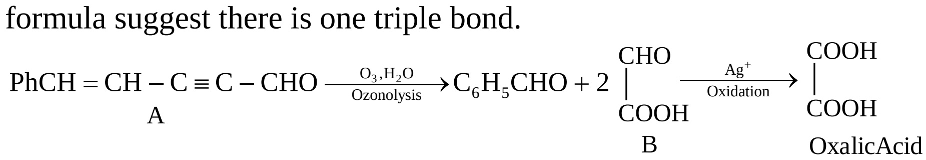
18. Ans. : A

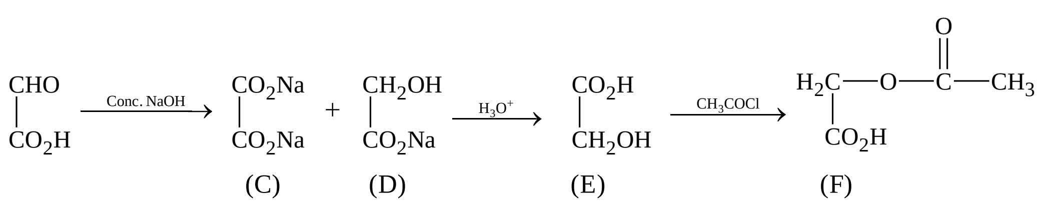
Sols. 16, 17, 18

A is an aldehyde which do not contain any α-hydrogen, so that it does not undergo self aldol condensation. Moreover, it must have PhCH =, moity as its ozonolysis gives benzaldehyde,

$$\text{DBE} = \frac{2\times 11 - 8 + 2}{2} = \frac{16}{2} = 8$$

Four for benzene ring, one for double bond adjacent to benzene ring. Examining the formula suggest there is one triple bond.





19. A –S; B – RT; C – Q, D – P

20. A – Q,R T ; B – R,T ; C – P ; D – P,R,T

MATHEMATICS : 21) C; 22) C; 23) D; 24) D; 25) B; 26) C; 27) C; 28) C; 29) AB; 30) ABC; 31) AB; 32) ACD; 33) B; 34) B; 35) A; 36) C; 37) D; 38) A; 39) A – Q; B – S; C – R, D – P; 40) A – S, B – Q, C – S, D – P.

25-01-2010 – MATHEMATICS – ANSWERS AND SOLUTIONS :

21. Ans. : C

Sol. $\because$ A + B + C = π and Δ is acute angled cot A, cot B, cot C all are positive.

Also, cot A cot B + cot B cot C + cot C cot A = 1 ….. (i)

$\because$ A.M. $\geq$ G.M.

$\therefore \frac{(1-\cot A \cot B)+1-\cot B \cot C)+(1-\cot C \cot A)}{3} \geq \sqrt[3]{(1-\cot A\cot B)(1-\cot B\cot C)(1-\cot C\cot A)}$

$\rightarrow \frac{3-1}{3}$

$\geq$ {(1 – cot A cot B) (1 – cot B cot C) (1 – cot C cot A)}$^{1/3}$ [from (i)]

$\therefore$ (1 – cot A cot B) (1 – cot B cot C) (1 – cot C cot A) $\leq \frac{8}{27}$.

22. Ans. : C

Sol. The two lines are $a\bar{z} + \bar{a}z + 1 = 0$ and $ib\bar{z} + i\bar{b}z - i = 0$ or $ib\bar{z} - (i\bar{b})z - i = 0$.

Since the lines are perpendicular to each other ib = a.

$\frac{a}{b}$ = i = $-\frac{\bar{a}}{\bar{b}} \rightarrow a\bar{b} + \bar{a}b = 0$

Hence (C) is the correct answer.

23. Ans. : D

Sol. $\frac{AB}{BC} = \sqrt{2}$

Considering the rotation about 'B' we get $\frac{z_1 - z_2}{z_3 - z_2} = \frac{|z_1 - z_2|}{|z_3 - z_2|} e^{i\pi/4} = \frac{AB}{BC} e^{i\pi/4} = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 1 + i.$

$\to z_1 - z_2 = (1 + i)(z_3 - z_2) = -iz_2$

$\to z_1 - (1 + i)z_3 = z_2(1 - 1 - i) = -iz_2$

$\to z_2 = iz_1 + i(1 + i)z_3 = -z_3 + i(z_1 + z_3).$

24. Ans. : C

Sol. $z_1 = a + ib,\ z_2 = c + id$

$|z_1| = |z_2| = 1 \to a^2 + b^2 = c^2 + d^2 = 1$

$\omega_1 = a + ic,\ \omega_2 = b + id$

$z_1 \bar{z}_2 = (a + ib)(c - id) = (ac + bd) + i(bc - ad)$

as $\text{Re}(z_1 \bar{z}_2) = 0 \to ac + bd = 0 \to ac = -bd$

We have $a^2 + b^2 = c^2 + d^2 \to a^2 - c^2 = d^2 - b^2$

$\to a^2 - c^2 + 2iac = d^2 - b^2 - 2ibd$ (as $ac = -bd$)

$\to (a + ic)^2 = (d - ib)^2 \to a + ic = (d - ib)$ or $-d + ib$.

$\to a = d$ and $c = -b$ or $a = -d.\ b = c$

$\to c^2 + d^2 = b^2 + d^2,\ a^2 + c^2 = a^2 + b^2.$

$\to a^2 + c^2 = 1,\ b^2 + d^2 = 1$

$\to |\omega_1| = |\omega_2| = 1$ also $ab + cd = -cd + cd = 0$

$\text{Re}(\omega \bar{\omega}_2) = 0.$

25. Ans. : B

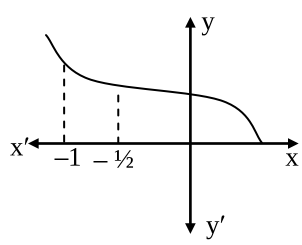
Sol. Given, $\cos^{-1}\left(\frac{n}{2\pi}\right) > \frac{2\pi}{3}$

$\to -1 < \frac{n}{2\pi} < -\frac{1}{2}$

$\to -2\pi < n < -\pi$

$\to -6.28 < n < 3.14$

$\to -6 \le n \le -4$ where $n \in 1$.



26. Ans. : C

Sol. $\cos^2 A + \cos^2 B - \cos^2 C = 1$

$\to 1 - \sin^2 A + 1 - \sin^2 B - 1 + \sin^2 C = 1$

$\to \sin^2 A + \sin^2 B = \sin^2 C$

$\to a^2 + b^2 = c^2$

Thus, triangle is right angled at c.

27. Ans. : C

Sol. Here, $\Delta = \frac{1}{2}a.\ AD$

$\to AD = \frac{2\Delta}{a} = \frac{abc}{b^2 - c^2}$ (Given)

$\to \frac{2\Delta}{a} = \frac{4R.\Delta}{b^2 - c^2} \to 2Ra = b^2 - c^2$

$\to \sin A = \sin^2 B - \sin^2 C$

$= \sin(B + C) . \sin(B - C)$

$= \sin A . \sin(B - C).$

$\to \sin(B - C) - 1 \to \angle B - \angle C = \frac{\pi}{2}$

$\to \angle B - \angle C = \frac{\pi}{2} \to \angle B = \frac{\pi}{2} + \angle C$

$= 90^o + 18^o = 108^o$

28. Ans. : C

Sol. $\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r^2}$

$= \frac{1}{\Delta^2}[(S - a)^2 + (S - b)^2 + (S - c)^2 + S^2]$

$= \frac{1}{\Delta^2}[4S^2 - 2S(a + b + c) + a^2 + b^2 + c^2]$

$= \frac{1}{\Delta^2}(a^2 + b^2 + c^2)$

$= \frac{2b^2}{\Delta^2} = \frac{2b^2}{\frac{1}{4}a^2c^2} = \frac{8b^2}{a^2c^2}$ $[\because b^2 = a^2 + c^2].$

29. Ans. : AB

Sol. L.H.S. and R.H.S. of the given equation are defined, if

$\frac{a-x}{a-b}$ = 0 and $\frac{x-b}{a-b}$ > 0

Either a > x > b or a < x < b

For a = x = b, LHS and RHS are not defined.

So options (A) and (B) are the correct answer.

30. Ans. : ABC

Sol. If we put x = tan θ, the given equality becomes $\tan^{-1} y = 4\theta$

$\rightarrow y = \tan 4\theta = \frac{2\tan 2\theta}{1-\tan^2 2\theta}$

$= \frac{2\left[\frac{2\tan\theta}{1-\tan^2 2\theta}\right]}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$

$= \frac{2\times 2(1-x^2)}{(1-x^2)^2-4x^2} = \frac{4x(1-x^2)}{1-6x^2+x^4}$

So that y is infinite, if

$x^4 - 6x^2 + 1 = 0$

$\rightarrow x^2 = \frac{6\pm\sqrt{36-4}}{2} = 3 \pm 2\sqrt{2}$

Hence options (A), (B) and (C) are correct.

31. Ans. : AB

Sol. It is clearly at x = $\frac{1}{2}$; a solution of the given equation which can be obtained by trial and error method. The given equation can be written as

$3\cos^{-1} x = \pi x + \frac{\pi}{2}$. ...... (i)

Since, the LHS of eq. (i) is a decreasing function and RHS of eq. (i) is an increasing function of x, the eq. (i) has only one solution. So x = $\frac{1}{2}$ is one and only one solution of the given equation.

32. Ans. : ACD

Sol. $\text{Re}\left(\frac{\tan\alpha - i(\sin\alpha/2+\cos\alpha/2)}{1+2i\sin\alpha/2}\right) = 0$

or $\text{Re}\left(\frac{\tan\alpha - i(\sin\alpha/2+\cos\alpha/2)(1-2i\sin\alpha/2)}{1+4\sin^2\alpha/2}\right)$

$\rightarrow \tan\alpha = 2\sin\alpha/2\ (\sin\alpha/2 + \cos\alpha/2)$

$\rightarrow \tan\alpha = 2\sin^2\alpha/2 + \sin\alpha \rightarrow \frac{\sin\alpha}{\cos\alpha} = \sin\alpha + 1 - \cos\alpha$

$\rightarrow \sin\alpha = \sin\alpha \,.\, \cos\alpha + \cos\alpha - \cos^2\alpha$.

$\rightarrow \sin\alpha(1-\cos\alpha) = \cos\alpha(1-\cos\alpha)$

$\rightarrow \sin\alpha = \cos\alpha,\ \cos\alpha = 1 \rightarrow \alpha = n\pi + \frac{\pi}{4},\ \alpha = 2n\pi\ (n \in 1)$

Hence (A), (C) and (D) are correct.

33. Ans. : B

Sol. D. E, F are feet of altitudes from A, B, C respectively.

D = π – 2A, E = π – 2B, F = π – 2C

EF = a cos A = R sin D → R = 2R′.

$\frac{\Delta'}{\Delta} = \frac{2(R')^2\sin D\sin E\sin F}{2R^2\sin A\sin B\sin C}$ = 2 cos A cos B cos C.

$= -\frac{1}{2}\left[1+\sum\cos 2A\right]$.

34. Ans. : B

Sol. $r' = 2R\sin\frac{D}{2}\sin\frac{E}{2}\sin\frac{F}{2}$

$\frac{r'}{R} = 2\cos A\cos B\cos C = -\frac{1}{2}\left[1+\sum\cos 2A\right]$

35. Ans. : A

Sol. $r'_1 = 2R\sin\frac{D}{2}\cos\frac{E}{2}\cos\frac{F}{2}$.

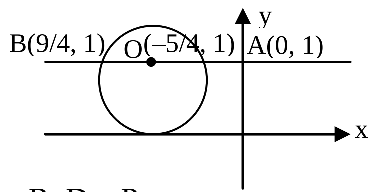
$\frac{r_1'}{R} = 2\cos A\sin B\sin C = \frac{1}{2}\left[1+\cos 2A-\cos 2B-\cos 2C\right]$.

36. Ans. : C
37. Ans. : D
38. Ans. : A

Sols. Clearly according to the least possibility $\alpha^2 - 7x + 11 \leq 1 \rightarrow \alpha \in [2, 5]$

The least value of $\alpha^2 - 7x + 11$ is $-\frac{5}{4}$

AB = $\frac{9}{4}$ and arg(z) = $\pi - \tan^{-1}(4/9)$ (point B)



39. Ans. : A – Q, B – S, C – R, D – P

Sol. A. In (0, cos 1), we have $\cos^{-1} x > \sin^{-1} x$

Also, $\cos^{-1} x > 1$ and $\sin^{-1} < 1$

$\rightarrow$ The greatest is $(\cos^{-1} x)^{\cos^{-1} x} = t_4$

and the least $(\sin^{-1} x)^{\cos^{-1} x} = t_2$ and $(\sin^{-1} x)^{\sin^{-1} x} < (\cos^{-1} x)^{\sin^{-1} x}$

$\rightarrow t_1 < t_3$

so, $t_4 > t_3 > t_1 > t_2$.

B. Similarly in $\cos 1 < x < \frac{1}{\sqrt{2}}$

$\cos^{-1} x > \sin^{-1} x$ and both are less than 1.

So, the greatest is t_3 and the least is t_2, also $t_4 > t_1$.

Hence, $t_3 > t_4 > t_1 > t_2$.

C. for $\frac{1}{\sqrt{2}} < x < \sin 1$

We have $1 > \sin^{-1} x > \cos^{-1} x$

So, the greatest is t_2 and the least is t_3, also $t_1 > t_4$.

Hence, $t_2 > t_1 > t_4 > t_3$

D. For $\sin 1 < x < 1$, we have

$\text{Sin}^{-1} x > \cos^{-1} x$

So, the greatest is t_1 and least is t_3 and $t_2 > t_4$.

Hence, $t_1 > t_2 > t_4 > t_3$.

40. Ans. : A – S, B – Q, C – S, D – P

Sol. A. $\angle BOD = \angle COD$

$\therefore \frac{z_4 - 0}{z_2 - 0} = e^{iA}$ and $\frac{z_3 - 0}{z_4 - 0} e^{iA}$

$\therefore z_4^2 = z_2 z_3 \rightarrow \frac{z_2 z_3}{z_4^2} = 1$

B. Clearly OD is perpendicular to BC, $\arg\left(\frac{z_4}{z_2 - z_3}\right)$.

C. $\angle COE = 2\angle CAE = 2\left(\frac{\pi}{2} - C\right)$

$\therefore \frac{z_3}{z_5} = e^{i(\pi - 2C)} = e^{i2C}$

Also, $\angle AOB = 2C \rightarrow \frac{z_2}{z_1} = e^{i2C}$.

$\therefore z_1 z_3 = z_2 z_5 \rightarrow \frac{z_1 z_3}{z_2 z_5} = 1$.

D. $\angle DOE = 2\angle DAE = 2\left\{\frac{A}{2} - \left(\frac{\pi}{2} - C\right)\right\} = A - \pi + 2C$

$\therefore \frac{z_5}{z_4} e^{i(A - \pi + 2C)} = e^{iA} \cdot e^{-i\pi} \cdot e^{2C} = \frac{z_4}{z_2}(-1)\frac{z_2}{z_1}$

$\rightarrow z_1 z_5 = -z_4^2 \quad \rightarrow \frac{z_4^2}{z_1 z_5} = -1$.

KEY : 41) C, 42) C, 43) B, 44) D, 45) D, 46) D, 47) C, 48) D, 49) ABC, 510) AD, 51) ABD, 52) ABD, 53) D, 54) C, 55) D, 56) D, 57) B, 58) C 59) A – PS, B – QT, C – RT, D – PS; 60) A – R, B – P, C – Q, D – P.

PHYSICS : SOLUTIONS :

41. Ans. : C

Sol. $dV_0 = \frac{\rho(4\pi r^2)dr}{4\pi\varepsilon_0 r} = \frac{\rho\pi dr}{\varepsilon_0}$

$V_0 = \frac{\rho}{\varepsilon_0}\int_R^{3R} r\,dr = \frac{\rho}{\varepsilon_0}\left(4R^2\right)$

$\rho = \frac{Q}{\frac{4}{3}\pi(27R^3 - R^3)} = \frac{3Q}{4\pi(26R^3)} \quad \therefore V_0 = \frac{3Q}{26\pi\varepsilon_0 R}$

42. Ans. : C

Sol. $dE_P = \frac{(\sigma dl)}{4\pi\varepsilon_0 (d+l)^2}\hat{i}$

$\overline{E}_P = \frac{\sigma\hat{i}}{4\pi\varepsilon_0}\int_0^{\infty}\frac{dl}{(x+l)^2} = \frac{\sigma\hat{i}}{4\pi\varepsilon_0}\left[-\frac{1}{x+l}\right]_0^{\infty} = \frac{\sigma}{4\pi\varepsilon_0 x}\hat{i}$

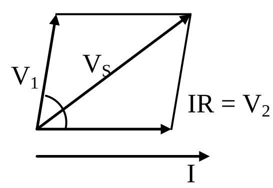
43. Ans. : B

Sol. $V_S^2 = V_1^2 + V_2^2 + 2V_1V_2\cos\theta$

Power developed across the coil

$= V_1 I\cos\theta$

$= V_1\left(\frac{V_2}{R}\right)\cos\theta = \frac{V_S^2 - V_1^2 - V_2^2}{2R}$.



44. Ans. : D

Sol. The current in the circuit $i = \left(\frac{V_0}{R}\right)e^{-\frac{Rt}{L}}$ $\therefore$ energy dissipated in R

$= \int_0^{\frac{2L}{R}} i^2 R\,dT = \frac{V_0^2 L}{2R^2}\left(\frac{e^4 - 1}{e^4}\right)$.

45. Ans. : D

Sol. The charge in side the surface S $= \frac{Q}{3}$

$\therefore$ flux $= \frac{Q}{3\varepsilon_0}$.

46. Ans. : D

Sol. $+V_0 - iR + \frac{V_0}{2} = 0, \quad i = \frac{3V_0}{2R}$.

47. Ans. : C

Sol. Steady state current $i_S = \frac{V_0}{3R}$

When S_2 is closed.

$V_0 - iR - (i - i_S)3R = 0 \qquad iR = \frac{V_0}{2}$

$\therefore V_b + V_0 - iR - 2i_S R = V_a$

$V_a - V_b = -\frac{V_0}{6}$.

48. Ans. : D

Sol. $E = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad r \geq 5R$

$U_e = \int_{5R}^{\infty}\left(\frac{1}{2}\varepsilon_0 E^2\right)4\pi r^2 dr = \frac{Q^2}{40\pi\varepsilon_0 R}$.

49. Ans. : ABC

Sol. $Q = \int_0^R \rho\left(4\pi r^2\right)dr = \frac{32\pi\rho_0 R^3}{15}$

$$E(4\pi r^2) = \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{r^3}{2}+\frac{r^5}{5R^2}\right) \qquad r \le R$$

$$E = \frac{\rho_0}{3\varepsilon_0}\left(r+\frac{3r^3}{5R^2}\right)$$

50. Ans. : AD

Sol. $E(2\pi r) = \frac{d}{dt}\left(B\pi R^2\right) = \pi R^2 \frac{dB}{dt} = \pi R^2\alpha$

$r \ge R.$

$E = \frac{R^2\alpha}{2r}$ At r = R $E = \frac{R\alpha}{2}$

For $r \le R$ $E(2\pi r) = \frac{d}{dt}\left(B\pi r^2\right)$

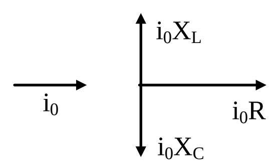
$E = \frac{r\alpha}{2}.$

51. Ans. : ABD

Sol. $V_{ab} = i_0(3R - R) = 2i_0R$ leading i by $\frac{\pi}{2}$

$$v_{ab} = 2i_0R\sin\left(\omega t+\frac{\pi}{6}+\frac{\pi}{2}\right)$$

$$= 2i_0R\sin\left(\omega t+\frac{2\pi}{3}\right)$$

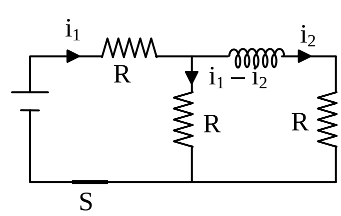


$$v_{cd} = \sqrt{2}i_0R\sin\left(\omega t-\frac{\pi}{2}\right)$$

$$\text{Power} = i_{rms}^2R = \left(\frac{i_0}{\sqrt{2}}\right)^2 R = \frac{i_0^{\,2}R}{2}$$

52. Ans. : ABD

Sol. t = 0 battery $i_0 = \frac{V_0}{2R}$



Power = $V_0 i_0 = \frac{V_0^{\,2}}{R}$

Maximum PD across L = $i_0R = \frac{V_0}{2}$

At time t

$V_0 - i_1R - (i_1 - i_2)R = 0$

$$V_0 - i_1R - L\frac{di_2}{dt} - i_2R = 0$$

$$\therefore \frac{di_2}{dt}+\frac{i_2}{\left(2L/3R\right)} = \frac{V_0}{2L}$$

$$i_2 = \frac{V_0}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right)$$

$$\in_L = l\frac{di_2}{dt} = \frac{V_0}{2}e^{-\frac{3Rt}{2L}}$$

At $t = \frac{2L}{R}$ $\in_L = \frac{V_0}{2}e^{-3} = \frac{V_0}{2e^3}.$

53. Ans. : D

Sol. When S_1 is opened, $90\%\left(\frac{1}{2}CV_0^{\,2}\right) = \frac{Q^2}{2C}$

$Q = \frac{3CV_0}{\sqrt{10}}$ charge flown through the battery = Q $\therefore$ battery work = $\frac{3CV_0}{\sqrt{10}}(V_0)$

Energy dissipated inR = $\frac{3CV_0^{\,2}}{\sqrt{10}} - \frac{9CV_0^{\,2}}{20} = \frac{CV_0^{\,2}}{10}\left(3\sqrt{10}-\frac{9}{2}\right).$

54. Ans. : C

Sol. LC oscillations $\frac{d^2i}{dt^2} = \omega^2 i$

$i = i_0 \sin \omega t$ (as i = 0 at t = 0)

$\frac{q}{C} - L\frac{di}{dt} = 0$ $q = LCi_0\omega \cos \omega t$

$= Q \cos \omega t.$

55. Ans. : D

Sol. Maximum energy in L = $\frac{9}{20}CV_0^2$

$\frac{1}{2}Li^2 = 50\%\left(\frac{9}{20}CV_0^2\right)$ i =

56. Ans. : D

Sol. $B(2\pi r) = \mu_0 i_0$ $B = \frac{\mu_0 i_0}{2\pi r}$

r > 2R $B(2\pi r) = \mu_0(i_0 - i_0) = 0$ B = 0.

57. Ans. : B

Sol. Energy density $\mu_m = \frac{B^2}{2\mu_0}$

$\mu_m = \int_R^{2R} \mu_m (2\pi r dr)l = \frac{\mu_0 i_0^2 l}{4\pi}\ln 2.$

58. Ans. : C

Sol. $0 \le r \le R$

$B = \mu_0\sigma_0\left(\frac{r}{2} - \frac{r^3}{8R^2}\right)$

$\frac{dB}{dr} = \mu_0\sigma_0\left(\frac{1}{2} - \frac{3r^2}{8R^2}\right)$

$\frac{dB}{dr} = 0$ for $r = \frac{2R}{\sqrt{3}} > R$

59. Ans. : A – PS, B – QT, C – RT, D – PS

Sol. Ampere's law $0 \le r \le R$

$B(2\pi r) = \mu_0 i'$

where $i' = \int_0^r \sigma_0\left(1 - \frac{r^2}{2R^2}\right)(2\pi r dr)$

$= 2\pi\sigma\left[\frac{r^2}{2} - \frac{r^4}{8R^2}\right]_0^r$

$= \pi\sigma_0\left(r^2 - \frac{r^4}{4R^2}\right)$

$B = \frac{\mu_0\sigma_0}{2}\left(r - \frac{r^3}{4R^2}\right)$

$\frac{dB}{dr} = 0$ at $r = \frac{2R}{\sqrt{3}}$ which is > R.

$\frac{dB}{dr} = \frac{\mu_0\sigma_0}{2}\left(1 - \frac{3r^2}{4R^2}\right) > 0$ and decreases with r.

$B_{r=R} = \frac{\mu_0\sigma_0}{2}\left(R - \frac{R}{4}\right) = \frac{3\mu_0\sigma_0 R}{8}.$

60. Ans. : A – R, B – P, C – Q, D – P.

Sol. $0 \le r \le R$ E = 0 $U_e = 0$

$R \le r \le 2R$ $E = \frac{Q}{4\pi\varepsilon_0 r^2}$

$2R \le r \le 3R$ $E = \dfrac{3Q}{4\pi\varepsilon_0 r^2}$

$r \ge 3R$ $E = 0,$ $U_e = 0$