KRISHNA MURTHY IIT ACADEMY

SOLUTIONS FOR UNIT-3 : PAPER-2

21ST JANUARY, 2010

SOLUTIONS AND ANSWERS FOR CHEMISTRY UNIT-3, PAPER-2, DT: 21-01-2010

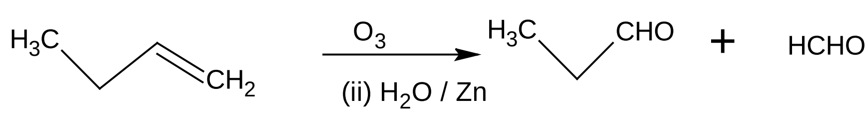
1. Ans. : A
Sol. protonation at ‘a’ will generate aromaticity in the molecule.

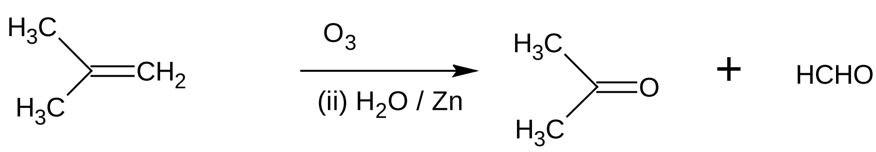
2. Ans. : D

3. Ans. : B

4. Ans. : C

5. Ans. : A, C,D
Sol.





6. Ans. : A,B,C

7. Ans. : A,B,D
Sol. all geometrical isomers are diastereomers of each other

8. Ans. : A, B, D
The strong acidic medium makes -N(CH_3) group meta director due to protonation.
Wurtz reaction disproportionation takes place so multiple products are formed

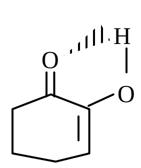
9. Ans. : CD

10. Ans. : A – Q, S; B – P, ; C – RT; D – RT

11. Ans. : A – P; B – S; C – Q; D – R

12. Ans. 6
Sol. $CH_3 - C \equiv C - C \equiv C - CH_2 - CH_3$.

13. Ans. : 8
Sol. Product contains 3 asymmetric carbon atoms.

14. Ans. : 2
Sol. 

enolic form.

15. Ans. : 8
Sol. x = 2, y = 6.

16. Ans. : 3
Sol. $CH_3CH_2CH_3$, CH_3CH_3, $CH_2 = CH_2$

17. Ans. : 4.

18. Ans. : 4
Sol. Probability factor of A = 5
PF of B = 7.6
PF of C = 6
PF of D = 3
$\therefore$ For each 6 moles of C, moles of A + moles D = 8
$\therefore$ For 3 moles of C = 4 moles A + D are formed.

19. Ans. : 3
Sol. Molacularity = 1
Order = 2.

MATHEMATICS

20. Ans. : D
Sol. $\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)\left(\frac{1}{z}+1\right) = 1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} + \frac{1}{xyz}$

Using AM ≥ GM, $\left(\frac{x+y+z}{3}\right)^3 \geq xyz \Rightarrow \frac{1}{xyz} \geq 27$

$\therefore 1 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx} + \frac{1}{xyz} \geq 1 + \frac{3}{\sqrt[3]{xyz}} + \frac{3}{\sqrt[3]{x^2y^2z^2}} + \frac{1}{xyz}$

$\geq \left(1+\frac{1}{\sqrt[3]{xyz}}\right)^3 = 64.$

21. Ans. : C
Sol. No. of games having both teams of couples = $9C_2$ = 36.
No. of games having exactly one couple in team = $9C_2 \times 7C_1 \times 2$ = 504.
Required number of games = 36 + 504 = 540.

22. Ans. : C
Sol. Let $x^2 + 4x + 4 = (x + 2)^2 = t$, then $t \geq 0 \ \forall \ x \in R$.
So that $f(t) = at^2 + bt + c = 0$ should have distinct non-negative roots for which
$\frac{-b}{a} > 0$ and $a f(0) > 0$
$\Rightarrow ab < 0$ and $ac > 0$.

23. Ans. : B
Sol. Let A denotes the event that P_1 wins in 3rd round and B denotes the event that P_2 wins in first but loses in 2nd round. Then required to find P(B/A).

$P(B/A) = \frac{P(B \cap A)}{P(A)}$. $P(A) = \frac{^{(8n-1)}C_{(n-1)}}{^{8n}C_n} = 1/8.$

$P(B \cap A)$ = (Prob. that both P_1 and P_2 win in 1st) × (Prob. P_1 winning and P_2 losing 2nd round) × (Prob. P_1 winning in 3rd round)

$= \frac{^{(8n-1)}C_{(4n-2)}}{^{8n}C_{4n}} \times \frac{^{(4n-2)}C_{(2n-1)}}{^{4n}C_{2n}} \times \frac{^{(2n-1)}C_{(n-1)}}{^{2n}C_n}.$

$= \frac{n}{4(8n-1)}$. Hence $P(B/A) = \frac{2n}{8n-1}$.

24. Ans. : ABCD
Sol. $AB = BA \Rightarrow (AB)^T = (BA)^T = A^TB^T = AB \Rightarrow AB$ is symmetric.
Also $AA^{-1} = I \Rightarrow (AA^{-1})^T = I^T = I \Rightarrow (A^{-1})^T A^T = I$
$\Rightarrow (A^{-1})^T = A^{-1} \Rightarrow A^{-1}$ is symmetric.
Similarly B^{-1} is also symmetric.
Also $(A^{-1} B^{-1})^T = (B^{-1})^T (A^{-1})^T = B^{-1} A^{-1} = (AB)^{-1} = (BA)^{-1} = A^{-1} B^{-1}$.
$\Rightarrow A^{-1} B^{-1}$ is symmetric.

25. Ans. : C.
Sol. Using Binomial theorem, given

$$(1+\cos x)^{-3} = \left(\sqrt{2}-1\right)^3 \cdot \frac{1}{\sin^3 x} = \left(\frac{\sqrt{2}-1}{\sin x}\right)^3 = \left(\frac{\sin x}{\sqrt{2}-1}\right)^{-3}.$$

$$\Rightarrow 1+\cos x = \frac{\sin x}{\sqrt{2}-1} \quad \Rightarrow \quad \left(\sqrt{2}-1\right) + \left(\sqrt{2}-1\right)\cos x = \sin x.$$

$\Rightarrow \left(\sqrt{2}-1\right)\cos x - \sin x = \left(1-\sqrt{2}\right)$. On simplification,

$x = 3\pi/4$.

26. Ans. : BC

Sol. Applying $C_1 \to C_1 - C_2$, the determinant is

$$\begin{vmatrix} 4 & (\tan\theta-\cot\theta)^2 & 1 \\ 4 & (\sin\theta-\operatorname{cosec}\theta)^2 & 1 \\ 4 & (\sec\theta-\cos\theta)^2 & 1 \end{vmatrix} = 0.$$

27. Ans. : ABD

Sol. Prob. That white face turns up at 1st throw = $\frac{1}{2} \cdot \frac{9}{12} + \frac{1}{2} \cdot \frac{3}{12} = \frac{1}{2}$

Prob. That first two throws yield while face is

$\frac{1}{2} \cdot \frac{81}{144} + \frac{1}{2} \cdot \frac{9}{144} = \frac{5}{16}$.

Prob. That first throw yields black face = $1 - \frac{1}{2} = \frac{1}{2}$.

Prob. That 3rd throw yields black face given white face turns at first two throws is

$$\frac{\frac{1}{2}\cdot\frac{81}{144}}{\frac{5}{16}} \cdot \frac{3}{12} + \frac{\frac{1}{2}\cdot\frac{9}{144}}{\frac{5}{16}} \cdot \frac{9}{12} = \frac{3}{10}.$$

28. Ans. : AD

Sol. Observe that $\alpha^2 = a^2bc$, $\beta^2 = b^2ca$, $\gamma^2 = c^2ab$ and $2b = a + c$.

29. Ans. : A – PQ, B – PS, C – R, D – PQ.

Sol. A. Discriminant is $4(ac + bd)^2 - 4((a + b)(c + d)(a - b)(c - d)$

$= 4(ad + bc)^2$, a perfect square.

B. Discriminant $= 4(a^2 - b^2)^2 - 4(a + b)^2(a - b)^2 = 0$.

C. Show that disc is negative

D. $x_1 + x_2 + x_1x_2 = -a$, $x_1x_2 + x_1^2x_2 + x_1x_2^2 = b$, $x_1^2x_2^2 = -c$

$\Rightarrow x_1x_2(1 - a) + c = b$ or $x_1x_2(1 - a - x_1x_2) = b$

$\Rightarrow x_1x_2(1 - a) + c = b$ or $x_1x_2 = \frac{b-c}{1-a}$ which is rational.

30. Ans. : A – QR, B – P, C – PS, D – Q.

Sol. A. Given that $P(\bar{A}) = P(A \cap B) + P(\bar{A})$, $P(B) = P(A) + P(A)$.

$\Rightarrow P(A \cap B) = 0$

$P(A \cup B) = P(A \cap B) + 3P(A) \Rightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B) + 3P(A)$

$P(A \cap B) \neq P(A) . P(B)$.

B. $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3} = P(A) + \frac{1}{6}$

$\Rightarrow P(A) = \frac{5}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \Rightarrow P(A) . P(B) = \frac{2}{3} . \frac{1}{2} = \frac{1}{3} = P(A \cap B)$

C. $P(A) = \frac{2}{4} = \frac{1}{2}$, $P(B) = \frac{2}{4} = \frac{1}{2}$, $P(A \cap B) = \frac{1}{4} = P(A) . P(B)$.

D. $$P\{A/(A \cup B)\} = \frac{P\{(A \cup B) \cap A\}}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B) - P(A \cap B)} = \frac{P(A)}{P(A) + P(B)}$$

$\Rightarrow P(A \cap B) = 0$.

31. Ans. : 3

Sol. The roots are $\frac{2k \pm \sqrt{4k^2 - 4(k^2-1)}}{2} = m \pm 1$.

Given $k - 1 > -2$, $k + 1 < 4 \Rightarrow -1 < k < 3$

$k = 0, 1, 2$..

32. Ans. : 0

Sol. Let f(x) = $\begin{vmatrix} (x+2)^2 & (x+3)^2 & (x+4)^2 \\ x & x^2 & x^3 \\ 1 & 2x & 3x^2 \end{vmatrix}$ = $a_0 + a_1x + ... + a_6x^6$

then $f'(x) = a_1 + 2a_2x + ... + 6a_6x^5$, then $f'(0) = a_6$.

Now $f'(x) = \begin{vmatrix} 2(x+2) & 2(x+3) & 2(x+4) \\ x & x^2 & x^3 \\ 1 & 2x & 3x^2 \end{vmatrix} + \begin{vmatrix} (x+2)^2 & (x+3)^2 & (x+4)^2 \\ 1 & 2x & 3x^2 \\ 1 & 2x & 3x^2 \end{vmatrix} + \begin{vmatrix} (x+2)^2 & (x+3)^2 & (x+4)^2 \\ 1 & x^2 & x^3 \\ 1 & 2 & 6x \end{vmatrix}$

then $f' = 0 + 0 + 0 = 0$. ∴ Coeff. of x is 0.

33. Ans. : 9

Sol. Using principle of inclusion and exclusion,

${}^nC_3 - n.\,{}^{(n-2)}C_1 + n = 30$

$\frac{n(n-1)(n-2)}{6} - n(n-2) + n = 30$

$\Rightarrow n\left(\frac{n^2-3n+2}{6} - n + 3\right) = 30$

Or $n^3 - 9n^2 + 20n - 180 = 0$. $\Rightarrow n = 9$.

34. Ans. : 5

Sol. The probability is m/n, where n is the number of positive integral solutions of $x_1 + x_2 + ... + x_5 = 101$, which is ${}^{100}C_4$ and m is the number of non-negative integral solutions of $(2y_1 + 1) + (2y_2 + 1) + (2y_3 + 1) + ... (2y_5 + 1) = 1$ or which ${}^{52}C_4$.

∴ Prob. = $\frac{{}^{52}C_4}{{}^{100}C_4} = \frac{221}{3201} \Rightarrow k = 221$.

35. Ans. : 5

Sol. $\alpha + \gamma = 4/a$, $\alpha\gamma = 1/a$, $\beta + \delta = 6/b$, $\beta\delta = 1/b$

$\Rightarrow \frac{\alpha+\gamma}{\alpha\gamma} = 4$, $\frac{\beta+\delta}{\beta\delta} = 6$ $\Rightarrow \frac{1}{\alpha}+\frac{1}{\gamma} = 4 = \frac{2}{\beta}$ and

$\frac{1}{\beta}+\frac{1}{\delta} = 6 = \frac{2}{\gamma}$

$\Rightarrow \alpha = 1, \beta = ½, \gamma = {}^1/_3, \delta = ¼$.

∴ a = 3, b = 8.

36. Ans. : 1

Sol. Use that $A A^T = I$ to get x + 2y + 4 = 0 and 2x – 2y + 2 = 0 to get x = –2, y = –1.

37. Ans. : 0

38. Ans. : 0

Sol. $b = \left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{200}\right) - \left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{100}\right)$

$= \left[\left(1+\frac{1}{3}+\frac{1}{5}+.....+\frac{1}{199}\right)+\frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{100}\right)\right] - \left(1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{100}\right)$

$= 1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....+\frac{1}{199}-\frac{1}{200} = a.$

∴ a – b = 0.

PHYSICS

39. Ans. : A

Sol. $x = x_0 \sin \omega t$ $\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{k}{m}}$

$\frac{2x_0}{3} = x_0 \sin \omega t$ $t = \frac{1}{\omega}\sin^{-1}\left(\frac{2}{3}\right)$.

∴ Time taken from extreme position to the wall = $\frac{T}{4} - \frac{T}{2\pi}\sin^{-1}\left(\frac{2}{3}\right)$

∴ Time period of motion = $\frac{T}{2} - \frac{T}{\pi}\sin^{-1}\left(\frac{2}{3}\right)$

$= \pi\sqrt{\frac{m}{k}} - 2\sin^{-1}\left(\frac{2}{3}\right)\sqrt{\frac{m}{k}}$

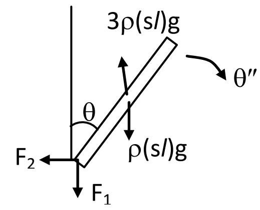
$= \sqrt{\frac{m}{k}}\left[\pi - 2\sin^{-1}\left(\frac{2}{3}\right)\right]$.

40. Ans. : C

Sol. $\left(\frac{l}{2}\theta\right)(Sl)g - 3\rho(sl)g\left(\frac{l}{2}\theta\right)$

$= \rho(Sl).\frac{l^2}{3}\theta''$

$\theta'' = \left(\frac{3g}{l}\right)\theta$; Time period T = $2\pi\sqrt{\frac{l}{3g}}$



41. Ans. : D

Sol. Kinetic energy is maximum when y = 0 for all x. cos ωt = 0, ωt = $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$.

42. Ans. : B

Sol. $\tan 135^o = \frac{v_P}{v}$

ω = 200π rad/s

k = 20π rad/m ∴ v = $\frac{\omega}{k}$ = 10.

∴ v_P = – 10 m/s

43. Ans. : BCD

Sol. Equilibrium position $k\partial_0 = mg$

∴ Amplitude A = $x_0 + \partial_0 = x_0 + \frac{mg}{k}$

Maximum elongation = $x_0 + 2\partial 0 = x_0 + \frac{2mg}{k}$

Maximum acceleration = $\frac{mg + kx_0}{m} = g + \frac{kx_0}{m}$

$= A\omega^2$.

44. Ans. : BCD

Sol. $l_1 = \frac{l}{3}$ Sh = Sh′ – $\left(\frac{l}{3}\right)l^2$

h′ – h = $\frac{l^3}{3S}$ $P_P' - P_P = \rho g(h' - h)$

$= \frac{\rho g l^3}{3S}$

45. Ans. : BC

Sol. $v_1 = (0.01)^2\pi = v_2(0.04)^2\pi$ $\frac{v_2}{v_1} = \frac{25}{4}$

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) = 10$

$Q = \rho v_1 \pi(0.1)^2 = 0.64 \times 10^{-3}\ \frac{m^3}{s}$.

46. Ans. : AC

Sol. Velocity of efflux v = $\frac{\sqrt{2gy}}{\sqrt{1-\left(\frac{a}{A}\right)^2}}$

Flow rate = va = $\frac{\sqrt{2gy}}{\sqrt{1-\left(\frac{a}{A}\right)^2}}a$

va = $\frac{d}{dt}(yA)$

dt = $-\frac{A}{a}.\frac{dy\sqrt{1-\left(\frac{a}{A}\right)^2}}{\sqrt{2gy}}$

$t_e = -\frac{A\sqrt{1-\left(\frac{a}{A}\right)^2}}{a\sqrt{2g}}\int_H^0 y^{-\frac{1}{2}}dy$ is independent of ρ.

$t_1 = = -k\int_H^{\frac{H}{2}} y^{-\frac{1}{2}} - k\left[2\sqrt{\frac{H}{2}} - 2\sqrt{H}\right]$

$t_2 = -k\int_{\frac{H}{2}}^{0} y^{-\frac{1}{2}} = -k\left[-2\sqrt{\frac{H}{2}}\right]$

$\frac{t_1}{t_2} = \frac{2-\sqrt{2}}{\sqrt{2}} = \sqrt{2}-1 = \frac{1}{\sqrt{2}+1}$

47. Ans. : ABC

Sol. $v = -\frac{a\pi}{2}\sin\frac{\pi t}{2}$

$v_{t=1} = -\frac{a\pi}{2}$ $x_1 - x_0 = 0 - a.$

$x_{10} - x_3 = -a - (0) = -a$

Distance traveled from t = 3 to t = 10 = 7a.

48. Ans. : A – P, B – S, C – R, D – Q

Sol. A. $-2k(R\theta)(R) = \frac{mg^2}{2}\theta''$

$\therefore \theta'' = -\left(\frac{4k}{m}\right)\theta$

$\therefore T = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$

B. $(2R\theta)\,k\,(2R) - mg(R\theta) = -\frac{3}{2}mR^2\theta''$

$\therefore \theta'' = -\left(\frac{8k}{3m} - \frac{2g}{3R}\right)\theta$ $\therefore T = \frac{2\pi}{\sqrt{\frac{8k}{3m} - \frac{2g}{3R}}}$ $k > \frac{mg}{4R}.$

49. Ans. : A – Q, B – R, C – S, D – T

Sol. $Mg - F_1 = M x''$

$F_1R - F_SR = (2M)\frac{R^2}{2}\alpha$

$F_1 = F_S = Mx''$ $(x'' = R\alpha)$

$F_S = k(\partial_0 + x)$ $k\partial_0 = Mg$

$\therefore x'' = -\left(\frac{k}{2M}\right)x$ $\therefore \omega_C^2 = \frac{k}{2M}$

$x = A\sin\omega_C t$ $A\omega_C = v_0$

$\therefore F_S = Mg + kA\sin\omega_C t$

$F_{hinge} = 2Mg + F_1 + F_S = 4mg + \frac{3}{2}kx$

$\therefore F_{S\,max} = Mg + KA = Mg + v_0\sqrt{2Mk}$

$F_{S\,min} = Mg - v_0\sqrt{2Mk}$

50. Ans. : 4.

Sol. Time period $T = 2\pi\sqrt{\frac{m}{4k}}$

Time taken $= \frac{3T}{4} = \frac{3}{4}2\pi\sqrt{\frac{m}{4k}}$

$= \frac{3\pi}{4}\sqrt{\frac{m}{k}}.$

51. Ans. : 9

Sol. $-(2kx)(2r) = \left(\frac{3}{2}mr^2 + m(2r)^2\right)\alpha$

$r\alpha = x''$

$$x'' = -\left(\frac{8k}{11m}\right)x$$

$$f = \frac{1}{2\pi}\sqrt{\frac{8k}{11m}} = \frac{1}{\pi}\sqrt{\frac{2k}{(2+9)m}}$$

52. Ans. : 3

Sol. $-\frac{d}{dt}(Ay) = \frac{2\sqrt{2gya}}{\sqrt{1-\left(\frac{a}{A}\right)^2}}$

$$\int_H^{H/9}\frac{dy}{\sqrt{y}} = -\frac{2\sqrt{2ga}}{\sqrt{1-\left(\frac{a}{A}\right)^2}}\int_0^T\frac{dt}{A}$$

$$T = \frac{\sqrt{\frac{2H}{g}\left(\frac{A^2}{a^2}-1\right)}}{3}$$

53. Ans. : 1

Sol. $y = y_0 \sin kx \cos \omega t$

$\lambda = \frac{2l}{4} = \frac{l}{2}$ $\qquad$ $k = \frac{2\pi}{\lambda} = \frac{4\pi}{l}$

$y = y_0 \sin \frac{4\pi x}{l} \cos \omega t$

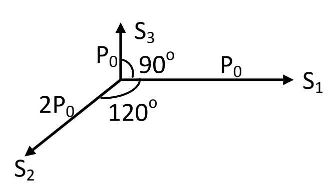
At $x = \frac{l}{8}$ $\qquad$ $A = y_0 \sin\frac{4\pi x}{l}$ $= y_0 \sin\frac{\pi}{2}$ $= y_0$.

54. Ans. : 3

Sol. Δx_1 = path different between S_1 and S_2 = $\frac{400\lambda}{3}$

$\Delta\phi_1 = \frac{2\pi}{\lambda}\,\frac{400\lambda}{3} = \frac{800\lambda}{3}$

$= 266\pi + \frac{2\pi}{3}$ $\qquad$ S_2 lagging



Resultant amplitude = $t_0\left(\sqrt{3}-1\right)$.

55. Ans. : 5

Sol. $f_{a_1} = f_0\left(\frac{v-v_0}{v-3v_0}\right)$

$$f_a^{1} = f_0\left(\frac{v}{v-3v_0}\right)$$

$$f_{a_2} = f_{a'}\left(\frac{v+v_0}{v}\right) = \frac{f_0(v+v_0)}{v-3v_0}$$

Beat frequency $f_b = f_{a^2} - f_{a^1}$

$$= \frac{f_0}{v-3v_0}(v+v_0-v+v_0)$$

$$= \frac{2f_0}{\frac{v}{v_0}-3} = \frac{2f_0}{193-3} = \frac{2f_0}{190} = \frac{f_0}{19(5)}$$

56. Ans. : 7

Sol. $-k\left(\frac{l\theta}{4}\right)\left(\frac{l}{4}\right)-k\left(\frac{3l}{4}\theta\right)\left(\frac{3l}{4}\right) = \left(\frac{7ml^2}{48}\right)\theta''$

$\theta'' = -\left(\frac{30k}{7m}\right)\theta$ $\therefore$ Time period $T = 2\pi\sqrt{\frac{7m}{30k}}$.

57. Ans. : 1

Sol. $T = 2\pi\sqrt{\frac{L}{2g}} = \pi\sqrt{\frac{2L}{g}}$

L = 4H + 2H = 6H

$T = \pi\sqrt{\frac{12H}{g}}$