

KRISHNA MURTHY



IIT ACADEMY

SOLUTIONS FOR UNIT-3 : PAPER-I

20th JANUARY, 2010

KEY – UNIT – 3 – PAPER – 1 – IIT(JEE) – 20-01-2010

CHE. : 1) C, 2) B, 3) C, 4) A, 5) A, 6) D, 7) D, 8) C, 9) AC, 10) BC, 11) ACD, 12) BC, 13) C, 14) D, 15) D, 16) D, 17) B, 18) D, 19) A – Q, B – QST, C – PR, D – Q; 20) A – RS, B – RS, C – PQS, D – PT.

MAT. : 21) A, 22) C, 23) C, 24) C, 25) C, 26) A, 27) B, 28) B, 29) ABCD, 30) AB, 31) AC, 32) AB, 33) B, 34) C, 35) B, 36) A, 37) C, 38) B, 39) A – S, B – R, C – Q, D – P; 40) A – R, B – P, C – S, D – Q.

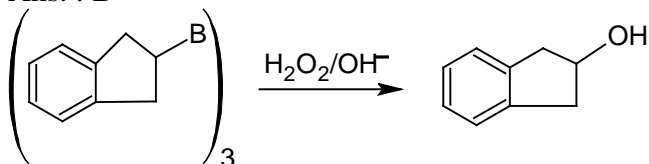
PHY. : 41) D, 42) C, 43) A, 44) B, 45) C, 46) C, 47) D, 48) B, 49) BC, 50) BCD, 51) AC, 52) CD, 53) D, 54) A, 55) C, 56) C, 57) A, 58) B, 59) A – S, B – P, C – Q, D – R; 60) A – R, B – R, C – P, D – Q.

CHEMISTRY – ANSWERS AND SOLUTIONS – 20-01-2010

1. Ans. : C

Sol. +R effect of CH₃ is higher than CH₃CH₂ – therefore order is 3>4>1>2

2. Ans. : B

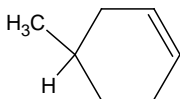


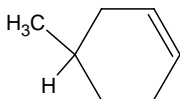
(Due to more stabilization of positive charge of benzene linked carbon.)

3. Ans. : C

4. Ans. : A

5. Ans. : A

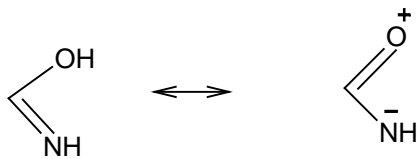


Product is , it has one chiral centre

6. Ans. : D

Sol. C₆H₅ - COOC₆H₅ + Br₂ → C₆H₅ - COOC₆H₄ (p - Br) ; the Oxygen linked benzene ring is more reactive.

7. Ans. : D



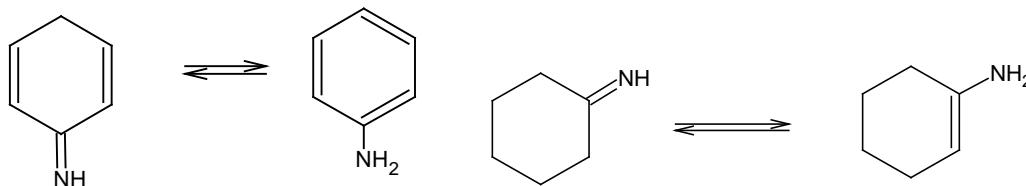
Sol. imidoformic acid

8. Ans. : C

Sol. CH₂N₂ → :CH₂ + N₂

9. Ans. : AC

10. Ans. : BC



Sol.

11. Ans. ACD

Sol. In AC,D Products are less stable than reactant.

12. Ans. : BC

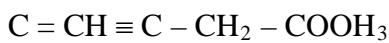
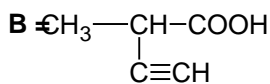
Sol. The reaction follows S_Ni mechanism

13. Ans. : C

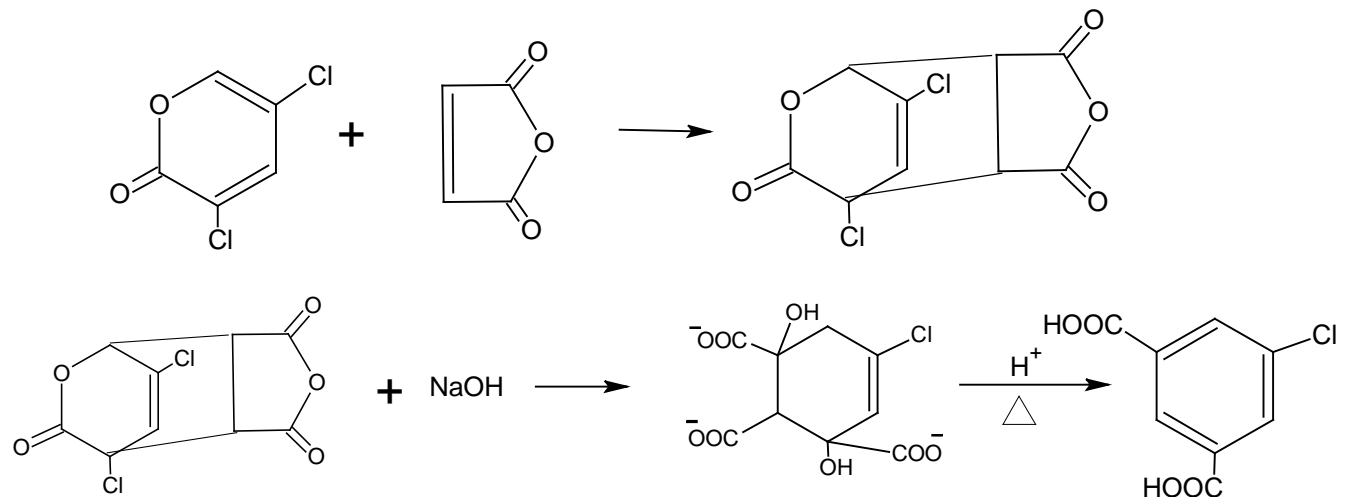
14. Ans. : D

15. Ans. : D

A = CH₃ - C ≡ C - CH₂COOH



16. Ans. : D



17. Ans. : B

Sol. since it is aromatic

18. Ans. : D

19. Ans. A – Q, B – S QT, C – RP, D – Q

20. Ans. : A – RS, B – SR, C – PQS, D – PT

MATHEMATICS – ANSWERS AND SOLUTIONS – 20-01-2010

21. Ans (a)

Sol. $(1-x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + {}^n C_n x^n$

On differentiating both sides w.r.t. x, we get

$$-n(1-x)^{n-1} = -{}^n C_1 + 2 {}^n C_2 x - 3 {}^n C_3 x^2 + \dots + (-1)^n n {}^n C_n x^{n-1} \dots (1)$$

and $(x+1)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} + {}^n C_2 x^{n-2} + \dots + {}^n C_n$

On differentiating both sides w.r.t. x, we get

$$n(x+1)^{n-1} = {}^n C_0 x^{n-1} + {}^n C_1 (n-1)x^{n-2} + {}^n C_2 (n-2)x^{n-3} + \dots + 2 {}^n C_{n-2} x + {}^n C_{n-1} \dots (2)$$

On multiplying equations (1) and (2), we get coefficient of x^n in $-n^2(1-x^2)^{n-1}$

$$= -n^2(1-x^2)^{n-1} 2 {}^n C_2 n {}^n C_0 - 3 {}^n C_3 (x-1) {}^n C_1 \dots + (-1)^n n {}^n C_n 2 {}^n C_{n-2}$$

$$= (-1)^{\frac{n-1}{2}} n^2 {}^{n-1} C_{n/2}$$

22. Ans (c)

Sol. Since $\alpha + i\beta$ is a root of the equation $x^3 + qx + r = 0$, $\alpha - i\beta$ is also its roots. Let the third root be γ , so that

$$\alpha + i\beta + (\alpha - i\beta) + \gamma = 0 \Rightarrow \gamma = -2\alpha \Rightarrow 2\alpha = -\gamma$$

Also γ is the root of the given equation.

Hence, 2α is one of the roots of the equation

$$(-x)^3 + q(-x) + r = 0 \Rightarrow x^3 + qx - r = 0$$

23. Ans : (c)

Sol. Since a_1, a_2, \dots are in A.P.

$$\therefore b_k = \frac{n}{2}(a_k + a_{n+k-1}) = \frac{n}{2}[a_1 + (k-1)d + a_1 + (n+k-2)d] = \frac{n}{2}[a_n + a_1 + 2(k-1)d]$$

$$\Rightarrow \sum_{k=1}^n b_k = \frac{n}{2} \left[na_n + na_1 + 2d \sum_{k=1}^n (k-1) \right] = \frac{n}{2} [na_n + na_1 + d(n-1)n]$$

$$= \frac{n^2}{2} [a_n + a_1 + (n-1)d] = \frac{n^2}{2} [a_n + a_n] = n^2 a_n$$

24. Ans: (c)

Sol. If A is the nth root of $I_2 \Rightarrow A^n = I_n$

$$\therefore A^n = \begin{bmatrix} a^n & na^{n-1}b \\ 0 & a^n \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow a^n = 1, b = 0$$

25. Ans: (c)

- Sol. If we put minimum number of balls required in each box. Balls left are $\frac{n(n-1)}{2}$ which can be put in $\frac{n^2+n-2}{2} C_{n-1}$ ways without restriction.
26. Ans: (a)
- Sol. First distribute right handed glove to four persons (1 glove to each person) in 4! Ways, then distribute left handed glove in such a way that no person gets a pair.
The number of favourable ways
 $= 4! \times 9 = 24 \times 9 = 219$
But total number of ways $= 4! \times 4! = 576 \therefore$ Required probability $= \frac{216}{576}$
27. Ans (b)
- Sol. The probability of winning of A the second race $= \frac{1}{2}$. Since winning of the first race and the second race are independent events.
28. Ans. (b)
- Sol. For maximum trace, all the diagonal elements should be equal to 1. The matrix is symmetric. \therefore The elements above the diagonal can be chosen in $3 \times 3 \times 3 = 27$ ways. After this, the element below the diagonal can be chosen only in one way.

Section 2: Multiple choice questions (More than one correct) (4 Questions)

29. Ans (a b c d)

Sol. $f(-1)f(0) < 0$

$$\Rightarrow (2a - b - c)(c + a - 2b) < 0$$

$$\Rightarrow (a - b + a - c)(c + a - 2b) < 0$$

$$\Rightarrow c + a < 2b$$

One root is 1 and the other is $\frac{c+a-2b}{a+b-2c} \therefore$ Both roots are rational.

Now, discriminant of $ax^2 + 2bx + c = 0$ is $4b^2 - 4ac$

Using $(c+a) < 2b$ $D > 0$

Also, a, b and c are +ve.

\therefore Both the roots are real and -ve.

30. Ans (a b)

Sol. $\Delta = 1 + \cos 2\beta$

31. Ans (a, c)

Sol. $\sum_{r=0}^n \frac{r}{{}^n C_r} = \sum_{r=0}^n \frac{n-r}{{}^n C_{n-r}} = \frac{n}{2} \sum_{r=0}^n \frac{1}{{}^n C_r}$

$$\Rightarrow \frac{n}{2} = \frac{n^2 - 3n + 3}{2}$$

$$\Rightarrow (n-1)(n-3) = 0$$

32. Ans: (a,b)

Sol. $ab + bc + ca - c^2 - 2ab$ are in A.P.

$$\Rightarrow (c-a)(a-b), (a-b)(b-c), (b-c)(c-a) \text{ are in A.P.}$$

Section 3 : Comprehension

Passage 1:

Size of the sample space, $n(S) = {}^n C_3 = \frac{n(n-1)(n-2)}{6}$

$n(E_1)$ = Number of triangles with exactly one side common $= n \cdot {}^{n-4} C_1 = n(n-4)$

$n(E_2)$ = Number of triangles with atleast one side common $= n \cdot {}^{n-2} C_1 - n = n(n-3)$

$n(E_3)$ = Number of triangles with no side common $= {}^n C_3 - n \cdot {}^{n-2} C_1 + n = \frac{n(n-4)(n-5)}{6}$

33. Ans. : B

Sol. Required probability $= \frac{n(E_3)}{n(S)} = \frac{(n-4)(n-5)}{(n-1)(n-2)}$

34 Ans. : C

Sol. Required probability = $\frac{n(E_1)}{n(S)} = \frac{6(n-4)}{(n-1)(n-2)}$

35 Ans. : B

Sol. Required probability = $\frac{n(E_2)}{n(S)} = \frac{6(n-3)}{(n-1)(n-2)}$

Passage 2:

A.adj A = $|A|I_n$ (where n is the order of the square matrix)

$\therefore |A| |adjA| = |A|^n |I_n| \Rightarrow |adjA| = |A|^{n-1}$

Similarly, $|adj(adjA)| = |adjA|^{n-1} = |A|^{(n-1)^2}$

$|adj(adj(adjA))| = |adj(adjA)|^{n-1} = |A|^{(n-1)^3}$

Also, adj A. adj(adjA) = $|adjA|I_n \Rightarrow adj(adjA) = |A|^{n-2} \cdot A$

adj (adj A) adj (adj (adjA)) = $|adj(adjA)I_n \Rightarrow adj(adj(adjA)) = |A|^{(n-1)(n-2)} adjA$

From the given information, $|A|=10$

36 Ans. : A

Sol. $|adjA| = |A|^{3-1} = |A|^2 = 100$

37 An. : C

Sol. $|adj(adjA)| = |adjA|^{3-1} = 100^2 = 10^4$

38 Ans. : B

Sol. $|adj(adj(adjA))| = |adj(adjA)|^{3-1} = (10^4)^{3-1} = 10^8$

Section 4: Match the following:

39. Ans. : A – S, B – R, C – Q, D - P

Sol. A) The integers divisible by 3 are 33 in numbers

The integers are divisible by 5 are 20 in numbers.

The integers divisible by 7 are 14 in numbers.

The integers divisible by 3 and 5 are 6 in numbers.

The integers divisible by both 3 and 7 are 4 in numbers.

The integers divisible by both 5 and 7 are 2 in numbers.

There are no integers divisible by all three.

Hence the sum of numbers divisible by 3 or 5 or 7 =

$$\frac{33}{2}(3+99) + \frac{20}{2}(5+100) + \frac{14}{2}(7+98) - \frac{6}{2}(15+90) - \frac{4}{2}(21+84) - \frac{2}{2}(35+70) = 2838$$

B) $a^2 + b^2 - c^2 = 1993c^2$

$$\frac{\cot C}{\cot A + \cot B} = \frac{\cos C \sin A \sin B}{\sin^2 C} = \frac{a^2 + b^2 - c^2}{2c^2} = 996 \therefore 600P = 150$$

C) If I is the length of the chord, r be the distance of the mid point of chord from the centre and a is radius of given circle.

$\therefore r = a \cos \theta$ and $I = 2a \sin \theta$

$$\frac{2}{3}2a < 2a \sin \theta < \frac{5}{6}2a \Rightarrow \frac{\sqrt{11}}{6}a < r < \frac{\sqrt{5}}{3}a$$

\therefore The given condition is satisfied if the mid point of the chord lies within the region

between the concentric circles of radius $\frac{\sqrt{11}}{6}$ and $\frac{\sqrt{5}}{3}$.

$$P = \frac{5}{9} - \frac{11}{36} = \frac{1}{4} \quad 1000P = 250$$

D) $\sum_{r=1}^n [(r+2)(r+2)! - (r+1)(r+1)!]$

$= (n+2)(n+2)! = 998.998!$

$n = 996.$

40.. Ans. : A – R, B – P, C – S, D - Q

- Sol. A) For $x \in \mathbb{R}^+$, $x + \frac{1}{x}$ has the minimum value 2, so that the minimum value of the expression $\left(x + \frac{1}{x} + 1\right)^n$ is 3^n . $\Rightarrow 3^n = 243 \Rightarrow n = 5$.
- B) Let $A = (a-c)^2 + (b-c)^2 + (b-d)^2 - (a-d)^2$
 $= 2[b^2 + c^2 - ac - bc - bd + ad] \Rightarrow b^2 = ac, c^2 = bd \text{ and } ad = bc$
 On putting these values in equation (1), we get $A = 0$
- C) Let the terms be $\frac{2}{r^2}, \frac{2}{r}, 2, 2r, 2r^2$. \therefore Product = 2^5
- D) Let a, b and c be the number of balls in three different boxes, then $a + b + c = 9$
 Number of solutions is ${}^{11}C_2 = 55$ ways include those ways in this
 a, b, c are same
 two of a, b, c are same
 all a, b, c are distinct
 Now a, b, c are same in exactly one way ($a = b = c = 3$). Also these are 12 ways out of 55 ways in which exactly two of a, b, c are same. These 12 will be counted as $\frac{12}{3} = 4$ ways (as 3, 3, 1 : 3, 1, 3 and 1, 3, 3 are same ways). Thus the total number of required ways is $1 + 4 + 7 = 12$.

PHYSICS – ANSWERS AND SOLUTIONS – 20-01-2010

41. Ans. D

Sol. $F - mg = ml(\theta_0 \omega_C)^2 \quad \omega_C^2 = \frac{9}{l}$
 $F = mg(1 + \theta_0^2)$.

42. Ans. : C

Sol. After collision $v = \frac{v_0}{2}, \quad \omega_C = \sqrt{\frac{k}{2m}}$
 Amplitude = maximum displacement from the new equilibrium position
 $= \frac{mg}{K} \sqrt{1 + \frac{Kv_0^2}{2mg^2}}$.

43. Ans. : A

Sol. If $\theta =$ small angular displacement from equilibrium position $-2k(r\theta)r = \frac{mr^2}{2} \theta''$

$$\therefore \theta'' = -\left(\frac{4K}{m}\right)\theta \quad \therefore \omega_C = \sqrt{\frac{4K}{m}}$$

At $t = 0, \quad \omega = \omega_0, \quad \theta'' = 0$

$$\therefore \omega = \omega_0 \cos\left(\sqrt{\frac{4K}{m}}t\right)$$

44. Ans. : B

Sol. $mx'' - f = ma$

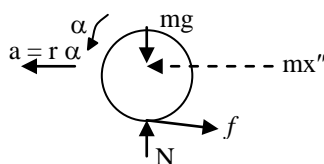
$$fr = \frac{2}{5} mr^2 \alpha$$

$$f = \frac{2}{5} ma$$

$$\therefore mx'' - f = \frac{5f}{2}$$

$$f = \frac{2mx''}{7} \quad |f_{\max}| = \frac{2}{7} mx_0 \omega^2$$

$$N = mg \quad \therefore \mu_s \geq \frac{2x_0 \omega^2}{7}$$



45. Ans. : C

Sol. $N - W - \rho_0 \pi R^2 = 0$

$$N = \left(\pi R^2 (3R) - \frac{2}{3} \pi R^3 \right) \rho g + \rho_0 \pi R^2$$

$$= \frac{7}{3} \pi R^3 \rho g + \rho_0 \pi R^2.$$

46. Ans. : C

Sol. $2\rho_{\text{sphere}} = \rho_{\text{liquid}} = 2\left(\frac{7}{8}\right)\rho = \frac{7\rho}{4}$

47. Ans. : D

Sol. Equation $y = y_0 \sin kx \cos \omega t$
 $\sin kl = \pm 1 \quad kl = \frac{7\pi}{2} \quad k = \frac{7\pi}{2l}$

$$\therefore y = y_0 \sin \frac{7\pi x}{2l} \cos \omega t$$

$$v_P = \frac{\partial y}{\partial t} = -y_0 \omega \sin \frac{7\pi x}{2l} \sin \omega t$$

$$\therefore v_{P \text{ max}} = y_0 \omega \sin \frac{7\pi x}{2l}$$

At $x = \frac{l}{3} \quad v_{P \text{ max}} = y_0 k v \sin \frac{7\pi}{6}$

$$= y_0 \frac{7\pi}{2l} \cdot v \left(\frac{1}{2} \right)$$

$$= \frac{7\pi y_0 v}{4l}.$$

48. Ans. : B

Sol. $S_2 : \rho = 2\rho_0 \sin\left(\omega t + \frac{\pi}{6} + \frac{\pi}{2}\right)$

$$= 2\rho_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$\therefore \rho_r^2 = \rho_0^2 + (2\rho_0)^2 + 2\rho_0(2\rho_0) \cos \frac{2\pi}{3}$$

$$= \rho_0^2 + 4\rho_0^2 - 2\rho_0^2 = 3\rho_0^2.$$

$\therefore I_r = 3I_0.$

49. Ans. : BC

Sol. $\omega = \sqrt{\frac{3k}{4m}} \quad F_{S \text{ max}} = \frac{3}{4} KA$

$$U_{S_1} = \frac{1}{2} k \left(\frac{3}{4} A \right)^2 = \frac{9}{32} KA^2$$

$$U_{S_2} = \frac{\left(\frac{3}{4} KA \right)^2}{2(3k)} = \frac{3KA^2}{32}.$$

50. Ans. : BCD

Sol. $mg R\theta - (2R\theta)k(2R)$

$$= \left(\frac{3}{2} mR^2 \right) \theta''$$

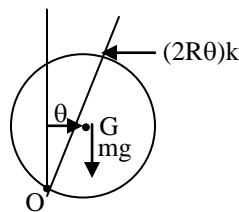
$$\theta'' = - \left(\frac{8k}{3m} - \frac{2g}{3R} \right) \theta$$

$$\therefore \omega_C = \sqrt{\frac{8k}{3m} - \frac{2g}{3R}}$$

Hinge reaction $F - mg = mR(\theta_0 \omega_C)^2$

$$= mR\theta_0^2 \frac{4\pi^2}{T^2}$$

$$F = mg \left[1 + \frac{4\pi^2 R \theta_0^2}{gT^2} \right]$$



51. Ans. : AC

Sol. $f = n\left(\frac{v}{4l}\right) \quad n = 1, 3, 5, 7, \dots$

$$f = 264 \text{ HZ} \quad v = 330 \text{ m/s}$$

$$\therefore \text{possible } l \text{ are } \begin{array}{ll} n = 1 & l = 31.25 \text{ cm} \\ n = 3 & l = 93.75 \text{ cm} \end{array}$$

$$n = 5 \quad l = 156.25 \text{ cm}$$

52. Ans. : CD

Sol. $\lambda = \frac{4l}{n} \quad n = 1, 3, 5, 7$

Third overtone $n = 7$

$$\frac{\lambda}{2} = \frac{2l}{7} = \frac{2}{7} (140 \text{ cm}) = 40 \text{ cm.}$$

53. Ans. : D }
54. Ans. : A }
55. Ans. : C }

Sols. $F = \frac{m}{l}(l-r)\left(\frac{l+r}{2}\right)\omega^2$
 $= \frac{m}{2l}(l^2 - r^2)\omega^2$
 $= F_{\max} = \frac{ml\omega^2}{2} = \sigma_b S$

$$\therefore \omega_{\max} = \sqrt{\frac{2\sigma_b S}{ml}}$$

$$\text{stress } \sigma = \frac{m\omega^2}{2lS}(l^2 - r^2)$$

$$\text{energy density } u = \frac{\sigma^2}{2Y} = \frac{m^2\omega^4}{4l^2S^2(2Y)}(l^2 - r^2)^2$$

$$\begin{aligned} \text{energy stored in segment } dr &= u(S \, dr) \\ &= \frac{m^2\omega^4}{4l^2S(2Y)}(l^2 - r^2)^2 dr \end{aligned}$$

$$\begin{aligned} \therefore \text{Elastic potential energy stored in the rod} &= \frac{m^2\omega^4}{8l^2SY} \int_0^l (l^2 - r^2)^2 dr \\ &= \frac{m^2\omega^4 l^3}{15SY}. \end{aligned}$$

56. Ans. : C }
57. Ans. : A }
58. Ans. : B }

Sols. For equilibrium $W = B$
Distance between CM and CB

$$= \frac{l}{2} - \frac{l}{3} = \frac{l}{6}$$

$$F + W = (\pi R^2 l) \left(\frac{3\rho}{2}\right) g$$

$$\begin{aligned} F &= \frac{3\rho}{2} \pi R^2 l g - (\pi R^2 l) \rho g. \\ &= \frac{\pi R^2 l \rho g}{2}. \end{aligned}$$

59. Ans. : A - S, B - P, C - Q, D - R.

Sol. $\omega_C = 12\pi = \sqrt{\frac{k}{m}}$
 $k = 144\pi^2(0.25) = 36\pi^2$
 $T = \frac{2\pi}{12\pi} = \frac{1}{6} \quad f = 6 \text{ Hz.}$

$$\therefore \frac{T}{2} = \frac{1}{12}$$

$$\begin{aligned} U_S + U_K &= 0 + U_{K \max} \\ &= \frac{1}{2} m(A\omega_C)^2 \\ &= \frac{1}{2} (0.25) 1^2 \times 144 \pi^2 \\ &= 18\pi^2. \end{aligned}$$

$$\therefore U_{S \max} = 18\pi^2.$$

60. Ans. : A - R, B - R, C - P, D - Q.

Sol. $\frac{\rho_0}{\rho} = \frac{\frac{1}{3}\pi H^3 - \frac{1}{3}\pi\left(\frac{H}{2}\right)^2\left(\frac{H}{2}\right)}{\frac{1}{3}\pi H^3} = 1 - \frac{1}{8} = \frac{7}{8}$

$$\rho_0 = \frac{7\rho}{8} = \text{density of the cone.}$$

$$\begin{aligned}\therefore \text{Weight of the cone} &= \frac{1}{3}\pi H^3 \left(\frac{7\rho}{8}\right) g. \\ &= \frac{7\pi H^3 \rho g}{24}\end{aligned}$$