Unit -2 paper -2 Solutions

Chemistry – ANSWERS AND SOLUTIONS

Answers and Solutions :

1. Ans. . A

Sol.

$$pH = pK_a + \log\left[\frac{CH_3COO^-}{CH_3COOH}\right] = 4.74 + \log\left(\frac{0.30}{0.05}\right)$$

$= 4.74 + \log 6 = 4.74 + \log 3 + \log 2 = 5.52$

□ pOH = 8.48

2. Ans. : (B)

Sol. $H_3PO_3 \; \Delta \; 2H^+ + PO_3^{2-}$ $\Delta_rH = ?$

$2H^+ + OH^- \; \Delta \; 2H_2O$; $\Delta_rH = -55.84 \times 2 = -111.68$

$-106.68 = \Delta_{ion}H - 55.84 \times 2$

$\Delta_{ion}H = 5$ kJ/mol

3. Ans. : D

Sol. log(23.5) = 1.37 & log (95) = 1.97

$$n = 1 + \frac{\log[(t_{1/2})_1/(t_{1/2})_2]}{\log(P_2/P_1)}$$

$$= 1 + \frac{1.37 - 1.96}{\log(1/2)}$$

n =3

4. Ans. **:** C

Sol. At the end of 5 hours ; $A_{t,1} = A_0/2$

at the end of 10th hours : $A_{t,2} = A_o/4$

$A_{t,2} - A_{t,1} = A_o/4 = 0.25\,A_0$

amount decayed = 0.25 mol ; moles of O_2 formed = 0.25/2 = 1/8

$W = -P\,\Delta V = -nRT = -(1/8) \times 300 \times 2$ Cal = 75 Cal

5. Ans. : ABD

6. Ans. : BC

7. Ans. : ABC

8. Ans. : A,B,D

Sol. more number of atoms in Compound, higher standard entropy of formation

9. Ans. : A,BD

Sol. for MnS $Q = \frac{[M^{2+}][H_2S]}{[H^+]^2} = 10^{-2}$; $Q < K_{eq}$ so no ppt

for ZnS , $Q > K_{eq}$ so ppt will form , similarly for CoS and PbS ,also $Q > K_{eq}$ so ppt will form

10. Ans. : A – P,Q; B – R; C – Q,T; D – P, R

11. Ans. : A - R ; B – Q ; C – T , R ; D – S

12. Ans. : 2

Sol. K_p changes only if T changes.

13. Ans. : 5

Sol. $K_c = \frac{K_f}{K_b}$; $K_b = 5 \times 10^{-4}$

14. Ans. : 0 (zero)

Sol. Use $r = -\frac{\Delta A}{\Delta t} = k\,[A]^x$;

15. Ans. : 2

Sol. $[OH^-] = (0.01 \times 2 \times 25 \times 10^{-3}) / (50 \times 10^{-3}) = 10^{-2}$; $p_{OH} = 2$

16. Ans. : 4

Sol. $m = m_o / 2^n$; $n = 18/3 = 6$

17. Ans. : 2

Sol. $C_{14}H_{10} \rightarrow C_{14}H_8O_2 + 4\,e^-$; therefore moles of electrons exchanged = (5.2/208) x 4 = 0.1

$C_6H_5NO_2 + 6\,e^- \rightarrow C_6H_5NH_2$; mass of nitrobenzene reduced = (0.1 / 6) x 120 gm = 2 gm

18. Ans. : 4

Sol. $t = \frac{2.303}{K}\log\left(\frac{100}{10}\right) = \frac{2.303}{0.693/1.2}\log 10 = 4$

19. Ans. : 6

Sol. $\ln K_s = \frac{n \times E^\circ \times 2.303}{0.059} = 6$

MATHEMATICS – ANSWERS AND SOLUTIONS

20. Ans. : B

Sol. Equation of the line $\frac{ax}{c-1} + \frac{by}{c-1} + 1 = 0$ has two independent parameters. It can pass through a fixed point if it contains only one independent parameter. So there must be one relation between $\frac{a}{c-1}$ and $\frac{b}{c-1}$ independent of a, b and c so that $\frac{a}{c-1}$ can be expressed in terms of $\frac{b}{c-1}$ and straight line contains only one independent parameter. The given relation can be expressed as $\frac{5a}{c-1} + \frac{4b}{c-1} = \frac{t-20c}{c-1}$. Now RHS be independent of c if t = 20.

21. Ans. : B

Sol. Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the points are P(a cos 2α, b sin α), Q(a cos 2β, b sin 2β), R(a cos 2γ, b sin γ). The equation of the chord PQ is $\frac{x}{a}\cos(\alpha+\beta) + \frac{y}{b}\sin(\alpha+\beta) = \cos(\alpha-\beta)$. If PQ passes through focus (ae, 0), then e =. $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$. Similarly, if the chord PR passes through the focus (–ae, 0), then –e = $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$.

Thus from (i) and (ii), we have $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = -\frac{\cos(\alpha-\gamma)}{\cos(\alpha+\gamma)}$

Apply componendo and dividendo, we get $\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)} = \frac{\cos(\alpha+\gamma)-\cos(\alpha-\gamma)}{\cos(\alpha+\gamma)+\cos(\alpha-\gamma)}$

$\rightarrow \frac{2\cos\alpha\cos\beta}{2\sin\alpha\sin\beta} = \frac{2\sin\alpha\sin\gamma}{2\cos\alpha\sin\gamma}$ $\rightarrow \tan\beta\tan\gamma = \cot^2\alpha$.

22. Ans. : C

Sol. As the lines joining common points of intersection must be equally inclined to axes, so, tan α = –tan β → α + β = π.

23. Ans. : C

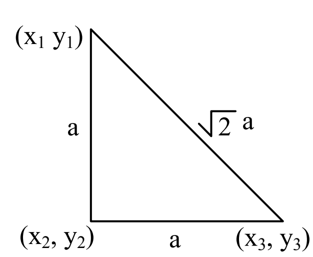
Sol. Let variable circle has centre at C and radius r. The two circles have centre C_1 and C_2 and radii r_1 and r_2 respectively. Then $C_1C = r_1 + r$, $C_2C = r_2 + r$.

∴ $CC_1 - CC_2 = r_1 - r_2$ is a constant.

∴ Locus of C is a hyperbola.

24. Ans. : ACD

Sol. Since a is an integer, $\sqrt{2}$ a is irrational now incentre will be



$$\left(\frac{ax_1+ax_2+\sqrt{2}ax_3}{2a+a\sqrt{2}},\frac{ay_1+ay_2+\sqrt{2}ay_3}{2a+a\sqrt{2}}\right)$$

Which is, clearly an irrational point.

25. Ans. : AC

Sol. $R=\left(1,\frac{1+3\lambda}{1+\lambda}\right)$. It is an interior point of $y^2-4x=0$

If $\left(\frac{1-3\lambda}{1+\lambda}\right)^2-4<0$.

Therefore, $-\frac{3}{5}<\lambda<1$. But $\lambda>0\rightarrow 0<\lambda<1$.

26. Ans. : BC

Sol. Clearly the vertices of the squares will lie on the director circle, i.e., on $x^2+y^2=4+3$ and hence the area of the squares is $2(4+3)=14$. Only one such square is possible.

27. Ans. : AB

Sol. We have $=\left|\sqrt{x^2+(y-1)^2}-\sqrt{x^2+(y+1)^2}\right|=k$.

Which is equivalent to $|S_1P-S_2P|=$ constant.

Where $S_1\equiv(0,1)$, $S_2\equiv(0,-1)$ and $P\equiv(x,y)$.

Using properties of a hyperbola, the above equation represents a hyperbola, then we have $2a=k$ [where 2a is the transverse axis and e is the eccentricity] and $2ae=S_1S_2=2$.

Dividing, we have $e=\frac{2}{k}$.

Since, $e>1$ for a hyperbola, therefore $k<2$

Also k must be a positive quantity.

Hence, we have, $k\in(0,2)$.

28. Ans. : AB

Sol. If two foci be S(5, 12) and S′ (24, 7) and it passes through origin O.

Then $SO=\sqrt{25+144}=13$; $S'O=\sqrt{576+49}=25$ and $SS'=\sqrt{386}$.

If conic be an ellipse, then $SO+S'O=2a$ and $SS'=2ae$.

$\therefore e=\frac{SS'}{SO+S'O}=\sqrt{\frac{386}{38}}$.

If conic be a hyperbola, then $S'O-SO=2a$ and $SS'=2ae$.

$\therefore e=\frac{SS'}{S'O+SO}=\sqrt{\frac{386}{12}}$

29. Ans. : A – P, B – R, C – S, D – Q

So,. The circle is $x^2+y^2-2x-4y-20=0$ and let the hyperbola be $xy=c^2$. If $\left(ct,\frac{c}{t}\right)$ be the points of intersection then $c^2t^2+\frac{c^2}{t^2}-2ct-\frac{4c}{t}-20=0$.

$\rightarrow c^2t^4-2ct^3-20t^2-4ct+c^2=0$.

If t_1, t_2, t_3, t_4 be its roots then,

$\Sigma t_1=\frac{2}{c}$; $\Sigma t_1t_2=-\frac{20}{c^2}$; $\Sigma t_1t_2t_3=\frac{4}{c}$ and $t_1t_2t_3t_4=1$.

A – $x_1+x_2+x_3+x_4=ct_1+ct_2+ct_3+ct_4=2$.

B – $\Sigma x_1x_2=c^2\ \Sigma t_1t_2=-20$.

$\therefore\ \Sigma x_1^2=(\Sigma x_1)^2-2\Sigma x_1x_2=44$.

C – $\Sigma y_1=\Sigma\frac{c}{t_1}=c\frac{\Sigma t_1t_2t_3}{t_1t_2t_3t_4}=4$.

$\Sigma y_1y_2=c^2\Sigma\frac{1}{t_1t_2}=-20$

$\therefore \Sigma y_1^2=(\Sigma y_1)^2-2\Sigma y_1y_2=56$.

D – $OA^2+OB^2+OC^2+OD^2=\Sigma(x_1^2+y_1^2)=100$.

30. Ans. : A – RS, B – S, C - P, D – PQ

Sol. A – Any point on the line is (t, 1 – t). The chord with this as mid point is

$y(1-t) - 2a(x+t) = (1-t)^2 - 4at$ it passes through (a, 2a)

$\rightarrow (1-t)^2 = 2a(1-a) > 0 \rightarrow 0 < a < 1 \quad \therefore$ L.R. $\in (0, 4)$.

B – The points P and Q are (1, 0) and (4, 0) and the circle is $(x-1)(x-4) + y^2 + \lambda y = 0$.

The length of the tangent from (0, 0) is $\sqrt{4} = 2$.

C – Solving the equation of the given lines, we get $x + a = 0$ so, the (0.1) tangents intersect at the directrix.

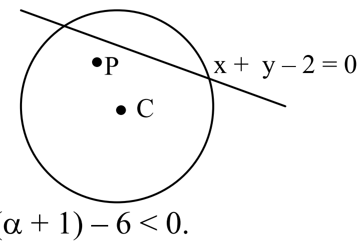
$\rightarrow m_1m_2 = -1$.

D – $1 - |h| > 0 \rightarrow -1 < h < 1$.

31. Ans. : 1

Sol. The given circle $S(x, y) \equiv x^2 + y^2 - x - y = 0$ …… (1)

has centre at $C \equiv \left(\frac{1}{2}, \frac{1}{2}\right)$



According to the required conditions, the given point $P(\alpha - 1, \alpha + 1)$ must lie inside the given circle.

i.e., $S(\alpha - 1, \alpha + 1) < 0$ i.e., $(\alpha - 1)^2 + (\alpha + 1) - (\alpha - 1) - (\alpha + 1) - 6 < 0$.

i.e., $\alpha^2 - \alpha - 2 < 0$ i.e., $(\alpha - 2)(\alpha + 1) < 0$.

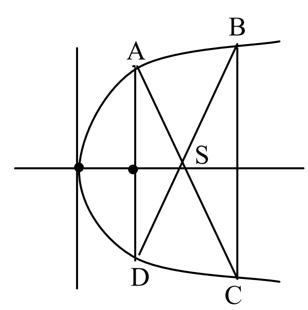
i.e., $-1 < \alpha < 2$ [Using sing – scheme from algebra. ….. (1)

32. Ans. : 5

Sol. Focus of the parabola $y^2 = 4x$ is (1, 0)

So diagonals are focal chord

$As = 1 + t^2 = c$ (say)



$\because \frac{1}{c} + \frac{1}{\frac{25}{4} - c} = 1 \quad \left[\because \frac{1}{AS} + \frac{1}{CS} - \frac{1}{a}\right]$

$\frac{25}{4} = \frac{25}{4}C - C^2$.

$\rightarrow 4c^2 - 25C + 25 = 0 = \rightarrow C = \frac{5}{4}, 5$.

For $C = \frac{5}{4}$. $1 + t^2 = 5 \rightarrow 1 + t^2 = 5 \rightarrow t = \pm 2$

$A \equiv \left(\frac{1}{4}, 1\right)$, $B \equiv (4, 4)$, $C \equiv (4, -4)$ and $D \equiv \left(\frac{1}{4}, -1\right)$

AD = 2 and BC = 8, distance between AD and BC = $\frac{15}{4}$

$\therefore$ Area of trapezium ABCD = $\frac{1}{2}(2 + 8) \times \frac{15}{4} = \frac{75}{4}$ sq. units.

33. Ans. : 7

Sol. Let $P(a\cos\theta_1; b\sin\theta_1)$, $Q(a\cos\theta_2, b\sin\theta_2)$ and $R(a\cos\theta_3, b\sin\theta_3)$ be the vertices of the triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The points on the auxiliary circle corresponding to these points are $P'(a\cos\theta_1, a\sin\theta_1)$, $Q'(a\cos\theta_2, a\sin\theta_2)$ and $R'(a\cos\theta_3, a\sin\theta_3)$.

$$\therefore \Delta_1 = \text{Area of } \Delta PQR = \frac{1}{2}\begin{vmatrix} a\cos\theta_1 & b\sin\theta_1 & 1 \\ a\cos\theta_2 & b\sin\theta_2 & 1 \\ a\cos\theta_3 & b\sin\theta_3 & 1 \end{vmatrix} = \frac{1}{2}ab\begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 1 \\ \cos\theta_2 & \sin\theta_2 & 1 \\ \cos\theta_3 & \sin\theta_3 & 1 \end{vmatrix}$$

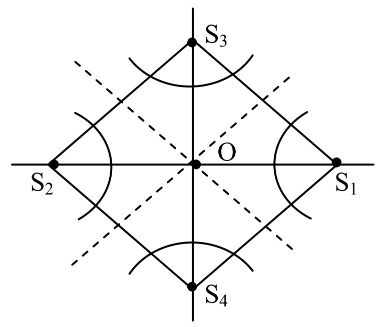
$$\text{and } \Delta_2 = \text{Area of } \Delta P'Q'R' = \frac{1}{2}\begin{vmatrix} a\cos\theta_1 & a\sin\theta_1 & 1 \\ a\cos\theta_2 & a\sin\theta_2 & 1 \\ a\cos\theta_3 & a\sin\theta_3 & 1 \end{vmatrix} = \frac{1}{2}a^2\begin{vmatrix} \cos\theta_1 & \sin\theta_1 & 1 \\ \cos\theta_2 & \sin\theta_2 & 1 \\ \cos\theta_3 & \sin\theta_3 & 1 \end{vmatrix}$$

Clearly, $\frac{\Delta_1}{\Delta_2} = \frac{a}{b} = \sqrt{1 - e^2} = 1/7$.

34. Ans. : 2

Sol. Due to summitry the desired area = 4 × area of

$\Delta S_1OS_3 = 4 \times \frac{1}{2}$ ae × be_1



Where e_1 is eccentricity of conjugate hyperbola

$= 2 \times 2e \times 3e_1 = 12ee_1$.

Now $b^2 = a^2(e^2 - 1) \rightarrow e^2 = 13/4$ and

$\frac{1}{e^2}+\frac{1}{e_1^2} = 1 \rightarrow e_1^2 = \frac{13}{9}$

$\therefore$ Required area $= 12 \times \frac{\sqrt{13}}{2} \times \frac{\sqrt{13}}{3} = 26$.

35. Ans. : 1

Sol. The equation of the tangent at P(a cos θ, b sin θ) on the given ellipse is

$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

Length of perpendicular from the focus (ae, 0)

$P = \frac{e\cos\theta - 1}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}} = \frac{ab(e\cos\theta-1)}{\sqrt{b^2\cos^2\theta + a^2(1-\cos^2\theta)}}$

$= \frac{ab\ e\cos\theta - 1}{\sqrt{a^2 - a^2e^2\cos^2\theta}} = -b\sqrt{\frac{1-e\cos\theta}{1+e\cos\theta}}$

$\rightarrow \frac{b^2}{p^2} = \frac{1+e\cos\theta}{1-e\cos\theta}$

Now $r^2 = (ae - a\cos\theta)^2 + b^2\sin^2\theta$

$= a^2[(e-\cos\theta)^2 + (1-e^2)\sin^2\theta)]$

$= a^2[e^2\cos^2\theta - 2e\cos\theta + 1] = a^2(1 - e\cos\theta)^2$

$\rightarrow r = a(1 - e\cos\theta)$

Now $\frac{2a}{r} - \frac{b^2}{p^2} = \frac{2}{1-e\cos\theta} - \frac{1+e\cos\theta}{1-e\cos\theta} = 1$.

36. Ans. : 2

Sol. Since (α, β) lies on the given line, $\frac{\alpha}{a}+\frac{\beta}{b} = 1$

$\rightarrow ab = a\beta + b\alpha \geq 2\sqrt{ab\alpha\beta}$ $\rightarrow 2S = ab \geq 4\alpha\beta$

$\rightarrow$ least value of $S = 2\alpha\beta$

37. Ans. : 0

Sol. Equation of normal at point 't' i.e. (ct, c/t) is;

$y - xt^2 = \frac{c}{t}(1-t)^4$ ...... (1)

If it meets the curve again 't' then $(ct_1, c/t_1)$ must satisfy (1)

$\rightarrow \frac{c}{t_1} - ct_1t^2 = \frac{c}{t}(1-t^4) \rightarrow \frac{1}{t_1} - t_1t^2 = \frac{1}{t_1} - t^3$

$\rightarrow \frac{1}{t_1} - \frac{1}{t} + t^2(t-t_1) = 0 \rightarrow \frac{(t-t_1)}{tt_1}(1 + t^3t_1) = 0$

Clearly $t \neq t_1 \rightarrow t^3t_1 + 1 = 0$.

38. Ans. : 4

Sol. The equation of a hyperbola of the series is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Where, λ is a parameter. The asymptotes of this hyperbola are $x/a = \pm y/\lambda$

Suppose (x′, y′) is a point p on the hyperbola which is equidistant from the transverse axis and an asymptote. Then $\frac{x'^2}{a^2} - \frac{y^2}{\lambda} = 1$ and $y' \frac{x'/a - y'/\lambda}{\sqrt{\frac{1}{a^2}+\frac{1}{\lambda^2}}}$

i.e., $\frac{y'^2}{\lambda^2} = \frac{x'^2}{a^2} - 1$ and $y'^2\left(\frac{1}{a^2}+\frac{1}{\lambda^2}\right) = \frac{x'^2}{a^2}+\frac{y'^2}{\lambda^2}-\frac{2x'y'}{a\lambda}$

The second relation gives on simplification,

$\left(y'^2 - x'^2\right)^2 = \frac{4x'^2y'^2a^2}{\lambda^2} = 4x'^2(x'^2 - a^2)$ by the first relation.

Hence, the focus of p is $(y^2 - x^2)^2 = 4x^2(x^2 - a^2)$

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39. Ans. : D

Sol. $W_{123} = NR(2T_0) + NRT_0 \int_0^{5cm} P\,A\,dx \;\; \ln\frac{P_1}{P_2} = -NRT_0 \ln 2 + 2\,NRT_0$

$U_3 - U_1 = N\left(\frac{5R}{2}\right)(2T_0) = 5NRT_0.$

$\therefore\ Q_{123} = NRT_0(7 - \ln 2)$

40. Ans. : C

Sol. $Q = \text{Heat transfer rate} = \frac{-2T_0}{\frac{1}{4\pi k}\left(\frac{1}{R}-\frac{1}{3R}\right)}$

$= -\,12\pi kRT_0$

$= -k(4\pi r^2)\,\frac{dT}{dr}$

$\therefore\ \frac{dT}{dr} = \frac{12\pi kRT_0}{4\pi k(2R)^2} = \frac{3T_0}{4R}.$

41. Ans. : C

Sol. $V - V_0 = \left(-\frac{V_0}{T_0}\right)(T - 2T_0) \qquad V = 3V_0 - \frac{V_0}{T_0}$

Work done $W_{ab} = \int_{V_0}^{2V_0} p\,dV = \int_{V_0}^{2V_0} \frac{NRT}{V}dV = NRT_0(3\ln 2 - 1).$

42. Ans. B

Sol. $W_{12} = -\,NRT_0 \ln 2, \quad Q_{12} = W_{12} < 0$

$W_{23} = NRT_0 \qquad Q_{23} = N\left(\frac{5R}{2}\right)T_0$

$W_{34} = 2NRT_0 \ln 2 \qquad Q_{34} \qquad W_{41} = -\,NRT_0$

$Q_{41} = -\frac{5NRT_0}{2} \qquad \therefore\ Q_{in} = \frac{5NRT_0}{2} + 2\,NRT_0 \ln 2.$

$\therefore$ efficiency $\eta = \frac{W_{cycle}}{Q_{in}} = \frac{\ln 2}{\frac{5}{2}+2\ln 2} = \frac{\ln 4}{5+\ln 16}.$

43. Ans. : C

Sol. $F_B = mg$ remains constant

$F_B = \rho V_i g \qquad \therefore\ V_i = \frac{m}{3\rho_0/(1+\gamma 4T)}$

$\therefore\ V_i$ increases. $\quad = \frac{m(1+\gamma 4T)}{3\rho_0} \quad (\gamma > 0)$

44. Ans. : ACD

Sol. Thermal resistance between r and ∞

$= \frac{1}{4\pi k_0 r} \qquad \therefore\ W = \frac{T - T_0}{\frac{1}{4\pi k_0 r}}$ under steady state.

$\therefore\ T = T_0 + \frac{W}{4\pi k_0 r} \qquad \frac{dT}{dr} = -\frac{W}{4\pi k_0 r^2}$

45. Ans. : ABC

Sol. $T_1 = T_2 = T_0$ $\therefore U_2 - U_1 = 0.$

$W_{12} = \frac{1}{2}(2V_0)(3P_0 + P_0) = 4 P_0V_0$

The straight line equation $P = 5P_0 - \frac{P_0V}{V_0}$

$PV = nRT = V\left(5P_0 - \frac{P_0V}{V_0}\right)$

For max. T $\frac{dT}{dV} = 0$ $\frac{d^2T}{dV^2} < 0$

$V = \frac{5V_0}{2}$ $T_{max.} = \frac{25T_0}{12}$

46. Ans. : BCD

Sol. Heat lost by container

$= \int_{300}^{500} m_C(A + BT)dT = 21600\ m_C$

Heat gained by ice = $mL + mC_W \Delta T$

= 10700 cal.

$\therefore 21600\ m_C = 10700$ $m_C = 0.5$ kg nearly.

47. Ans. : ABC

Sol. $P_i = P_0 + \frac{mg}{A} = 1.01 \times 10^5 \frac{N}{m_2}$

$P = P_0 + \frac{mg}{A} + \frac{kx}{A}$

Work done by the gas = $\int_0^{5cm} P\ A\ dx = 25.375$ J

First law $Q - W = 4U = 50 - 25.375$

$= nC_V \Delta T$

$\Delta T = 2$ nearly.

48. Ans. : A – RST, B – Q, C – QT, D – PS

Sol. ab, cd are isothermal processes $\therefore \Delta U = 0$

da is a constant volume process W = 0.

49. Ans. : A – R, B – P, C – T, D – P

Sol. For the tube $\Delta U = 0$

$\Delta U_l + \Delta U_r = 0$ $\frac{2 - \ln 4}{5}\ \frac{3}{2} n_1R(T_f - T_0)$

$+ \frac{3}{2} n_2R(T_f - 5T_0) = 0$

$5P_0V_0 = n_1RT_0$ $P_0V_0 = n_2R(5T_0)$

$\Delta U_r = \frac{3}{2} n_2R(T_f - 5T_0) = -\frac{15P_0V_0}{26}$

50. Ans. : 7

$PV = 2R\frac{kP_0T_0}{P}$ $\therefore PV^{0.5}$ = constant.

$W_{12} = \frac{2RT_0(k-1)}{1-0.5} = 4RT_0(k - 1)$

$U_2 - U_1 = \frac{2R}{(2/3)}(k - 1)T_0 = 3R(k - 1)T_0$

$\therefore Q_{12} = 7RT_0(k - 1).$

51. Ans. : 4

Sol. Work done $W_{ab} = 20$ J, $W_{cd} = -40$ J

$\therefore W_{cycle} = -20$ J $\qquad \therefore Q_{cycle} = -20$ J .

$\therefore$ Heat rejected = 20 J = 5(4) J.

52. Ans. : 8

Sol. $\frac{P_0V_0}{4} = NRT_1 \qquad \frac{3P_0V_0}{4} = NRT_2$

$\Delta U = 0 \qquad NC_V(T_f - T_1) + NC_v(T_f - T_2) = 0$

$\therefore T_f = \frac{T_1 + T_2}{2} = \frac{P_0V_0}{2NR}$

$Q = NC_v(T_f - T_1) = N\left(\frac{5R}{2}\right)\left(\frac{T_2 - T_1}{2}\right) = \frac{5P_0V_0}{8}$

53. Ans. : 6

$Q_1 = \frac{k_1A(80-40)}{0.2}, \qquad Q_2 = \frac{k_2A(80-40)}{0.2}$

$\therefore Q_1 + Q_2 = \frac{A(40)}{0.2}(k_1 + k_2) = \frac{0.2\times10^{-4}(40)(600)}{0.2}$

$= 2.4$ watt $= 0.4(6)$ watt.

54. Ans. : 4

Sol. $Q_{ab} = RT_0 \ln 3$

$Q_{bc} = (4R)(2T_0) = 8RT_0$

$\therefore$ molar specific heat $C = \frac{8RT_0 + RT_0\ln 3}{2T_0}$

$= R\left(4 + \frac{\ln 3}{2}\right) = R\left(4 + \ln\sqrt{3}\right)$

$\therefore k = 4.$

55. Ans. : 8

Sol. $\frac{U}{\rho}$ = constant $\qquad \therefore VT$ = constant

PV^2 = constant $\qquad W_{ab} = nR(T_a - T_b)$

$U_b - U_a = n\frac{R}{\gamma - 1}(T_b - T_a) = 2U_0$

$\therefore nR(T_b - T_a) = 2U_0\left(\frac{7}{5} - 1\right) = \frac{4U_0}{5} \qquad \therefore W_{ab} = -\frac{4U_0}{5}$

56. Ans. : 2

Sol. $Q = -kS\frac{dT}{dx}$

$\frac{Q\,dT}{Sk_0\left(1 + \frac{x^2}{l^2}\right)} = -dT$

$\frac{Ql^2}{Sk_0}\frac{1}{l}\left[\tan^{-1}\frac{x}{l}\right]_0^l = -\int_{10T_0}^{T_0} dT$

$\frac{Ql}{Sk_0}\left(\frac{\pi}{4}\right) = 9T_0 \qquad Q = \frac{36Sk_0T_0}{\pi l}$

$= \left(\frac{18k_0ST_0}{\pi l}\right)(2).$

57. Ans. : 2

Sol. $W_{ab} = nR(T_b - T_c) = nRT_0$

$Q_{ab} = n\left(\frac{5R}{2}\right)(T_0) = \frac{5}{2}nRT_0$

$W_{bc} = 0, Q_{bc} < 0 \qquad W_{ca} = nRT_0 \ln\left(\frac{1}{2}\right)$

$\therefore$ efficiency $\eta = \frac{nRT_0(1 - \ln 2)}{\frac{5}{2}nRT_0}$

$$= \frac{2}{5}(1 - \ln 2)$$

$$= \frac{2 - \ln 4}{5}.$$