KRISHNA MURTHY
IIT ACADEMY

SOLUTIONS FOR MODEL TEST FOR IIT(JEE)-2010

UNIT – 1 :: PAPER – II

5TH JANUARY, 2010

Time : 3 Hrs **Max. Marks : 240**

05-01-2010 – Unit – 1 – IIT-JEE – PAPER – II – ANSWERS :

CHEMISTRY : 1) D, 2) A, 3) C, 4) C
5) ABC, 6) ABD, 7) AC, 8) BC, 9) ABC
10) A – P, B – Q, C – R, D – S; 11) A – P, B – T, C – R, D – Q
12) 3; 13) 2; 14) 8; 15) 4; 16) 1; 17) 3; 18) 6; 19) 6.
MATHEMATICS : 20) A, 21) B, 22) D, 23) A
24) BC, 25) C, 26) ABC, 27) AD; 28) A
29) A – S, B – R, C – Q, D – P; 30) A – S, B – P, C – Q, R – R.
31) 2; 32) 4; 33) 1; 34) 4; 35) 2; 36) 3; 37) 4; 38) 8
PHYSICS : 39) C, 40) A, 41) D, 42) C
43) ABCD, 44) CD, 45) BCD, 46) BCD, 47) BCD
48) A – S, B – S, C – PS, D – Q; 49) A – P, B – P, C – P, D – QR
50) 1; 51) 2; 52) 7; 53) 9; 54) 4; 55) 9; 56) 5; 57) 8.

CHEMISTRY : ANSWERS AND SOLUTIONS :

1. Ans. : D

$$\frac{V/220}{V/200} = \sqrt{\frac{d_{O_2}}{d_{mix}}} \Rightarrow d_{mix} = 1.936 g/L$$

Let deMass of mixture = mass of O_3 + mass of O_2

$100 \times 1.936 = 40 \times d + 60 \times 1.6$ or $d = 2.44$ g/L

2. Ans. : A

Meq of FeO = Meq of $KMnO_4$

$= 0.25 \times 5 \times 100$

m.mole of FeO (n = 1) = $\frac{0.25 \times 100 \times 5}{1}$ = 125

Total m.eq or m.mole of Fe^{+2} = $1000 \times 0.1 \times 6 = 600$

m.mole of Fe^{+2} from Fe_2O_3 = 600 – 125 = 475

m.mole of Fe_2O_3 = $\frac{475}{2}$

wt. of FeO = $\frac{125 \times 72}{1000} = 9gm$

wt. of Fe_2O_3 = $\frac{475}{2} \times \frac{160}{1000} = 38gm$

% Fe_2O_3 = $\frac{38}{38+9} \times 100 = 80.85\%$

3. Ans. : C

Moles of NaOH consumed when reacted with H^+ = 3×1

Moles of H_2SO_4 present in sample = 1.5

Wt. of H_2SO_4 in sample = $98 \times 1.5 = 147$ gm

% purity = $\frac{149}{183.75} \times 100 = 80\%$

4. Ans. : C

5. Ans. : ABC

6. Ans. : ABD

7. Ans. : A,C

$$\frac{d\left[U(r)\right]}{dr} = \frac{3Ke^2}{r^4} \Rightarrow \text{ magnitude of force}$$

$$\frac{3Ke^2}{r^4} = \frac{mv^2}{r}, \text{ and } mvr = n\frac{h}{2\pi} \Rightarrow r = \frac{nh}{2\pi mv}$$

$$v = \frac{n^3h^3}{24Ke^2\pi^3m^2}$$

8. Ans. : BCD

Sol. Possible maximum number different spectral lines are = $5 \rightarrow 4$

$4 \rightarrow 3$

$3 \rightarrow 2$

$2 \rightarrow 1$ (Number = 4)

Since only one He^+ ion is considered.

$$\frac{1}{\lambda} = R_H . z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

for λ_{max}., $n_2 = 5$, $n_1 = 4$; $\frac{1}{\lambda_{max}} = R_H . z^2\left[\frac{1}{4^2} - \frac{1}{5^2}\right]$

$$\frac{1}{\lambda_{max}} = R_H . z^2\left(\frac{9}{400}\right)$$

$$\frac{1}{\lambda_{max}} = \left(\frac{9 \times R_H}{100}\right)$$

$$\lambda_{max.} = \frac{100}{9R_H} < \frac{10}{R_H}$$

$$r_n = \frac{n^2}{z} . r_0 = \frac{5^2}{2} \times 0.53A^o = 6.62A^o.$$

$$E_n = -\frac{z^2}{n^2} \; E_0 = -\frac{2^2}{5^2} \times 13.6 = -2.176eV$$

I E = 2.17eV > 2eV.

9. Ans.: A,B,C

Semi permeable membrane allows passage of only solvent not the solute.

10. Ans. : A – P, B – Q, C – R, D – S

11. Ans. : A – P, B – T, C – R, D – Q.

Sol. PV = nRT = k; $\therefore \frac{dV}{dp} = -\frac{k}{p^2}$

$$= \frac{(dV/dp)_T}{V} = \frac{k/p^2}{(k/p)} = \frac{1}{p}$$

Y = PV^2 = (k . V)

Y = K V (straight line with positive slope)

$$K E = \frac{3}{2} R (t^o C + 273)$$

12. Ans. : 3

$\Delta T = I\ k_f\ m$.

$0.7 = i \times 5 \times \frac{0.2 \times 1000}{20 \times 60}$; i = 0.84

2 RCOOH ⇌ $(RCOOH)_2$.

$i - \alpha \quad \alpha/2$

$$i = \frac{1 - \alpha/2}{1} = 0.84$$

$\alpha = 0.3 = 3 \times 10^{-3}$

13. Ans. : 2

14. Ans. : 8

15. Ans. : 4

$\frac{r_{CH_4}}{r_{He}} = \frac{80}{20}\left(\sqrt{\frac{4}{16}}\right)^n$; where n are number of diffusion steps ;

$\frac{20}{80} = \frac{80}{20}\left(\frac{1}{2}\right)^n$; n = 4

16. Ans. : 1

Total Moles of H_2SO_4 = 1.18 x 75 x 0.227/98 = 0.20 moles

Reacted moles of H_2SO_4 = 0.15 moles

Left moles of H_2SO_4 = 0.05 moles ;

Concentration = 0.05 / 0.05 = 1 M

17 Ans. : 3 ; $E_n = - z^2$ x 1312 KJ/mole = - 11180 KJ/mole
Z = 3

18. Ans. : 6 ; use KE = 3/2 nRT

19. Ans. : 6

MATHEMATICS – ANSWERS AND SOLUTIONS :

SECTION 1: Multiple choice questions (Single option correct) (4Questions)

20. Ans. : A

$$\int \frac{dx}{x^r(1+x^r)^{1/r}} = \frac{1}{(1-r)}(1+x^{-r})^{1-\frac{1}{r}} + c$$

On comparing, r = 3

$$\Rightarrow \alpha = -\frac{1}{2}, \beta = \frac{2}{3}$$

$$\Rightarrow 6x^2 - x - 2 = 0$$

21. Ans. : B

$$\frac{\lim\limits_{y\to\infty}\int_1^y [\tan^{-1} x]dx}{\lim\limits_{y\to\infty}\int_1^y \left[1+\frac{1}{x}\right]dx} = \lim_{y\to\infty}\frac{y-\tan 1}{y-1} = 1$$

22. Ans. : D :

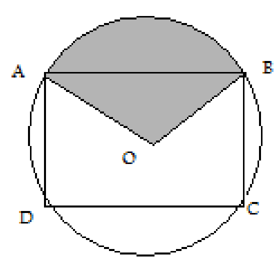
Let $f(x) = \sin(\sin 3x + 4\sin^{-3} x) = \sin(3\sin x)$

$\Rightarrow 3 \le 3\sin x \le 3 \Rightarrow f(x)_{\max} = 1$

23. Ans. : A

Shaded area is the required region

$$= \frac{\pi r^2}{4} = \frac{\pi(4)^2}{4} = 4\pi$$



SECTION 2: (Multiple choice questions) (mc stions)

24. Ans. : BC

Let $f(x) = \frac{\ln(\ln x)}{\ln x}$

$$f'(x)\frac{1-\ln(\ln x)}{x(\log x)^2} < 0 \,\forall\, x > e^e$$

$\Rightarrow f(x)$ is increasing in $(1, e^e)$ and decreasing in (e^e, ∞)

$\Rightarrow \; x > y \Rightarrow (\ln x)^{\ln y} < (\ln y)^{\ln x} \forall x, y \in (e^e, \infty)$

and $x < y \Rightarrow (\ln x)^{\ln y} < (\ln y)^{\ln x} \forall x, y \in (1, e^e)$

25. Ans. : C

RHL(at x = 1) = $\lim\limits_{h\to 0}(1+h)[1+h] = 1$

LHL (at x = 1) = $\lim\limits_{h\to 0}(1-h)[1-h] = 0$

$\Rightarrow f$ is discontinuous at x = 1.

Similarly, at x = 2, f(x) = x, for 1 < x < 2 and f(x) = 2 (x – 1) for 2 < x <3.

$f(2^-) = f(2^+) = 1$; but $f'(2^-) = 1$ and $f'(2^+) = 2$

26. Ans. : ABC

$f'(x) = -\cos x + a, if\ a > 1, then\ f(x) = 0$ has only one real root, which is positive if $f(0) < 0\ and\ positive\ if\ f(0) > 0$.

27. Ans. : AD.

Since $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a}[f(x)]$. So, this can occur only when $\lim\limits_{x\to a} f(x)$ is an integer.

$\lim\limits_{x\to a}[f(x)]$ can exist only when $f(x)$ is either minimum or maximum at the point x=a.

Since f is continuous function also $\lim\limits_{x\to a}[f(x)] = f(a)$.

$\therefore \; f(a)$ local minimum.

28. Ans. : A

Here $\frac{dy}{dx} = \frac{y\,(x - y\ln y)}{x\,(x\ln x - y)}$

$$\Rightarrow x^2 \ln x\,dy - xy\,dy = xy\,dx - y^2 \ln y dx$$
$$\Rightarrow \frac{\ln x}{y^2}dy - \frac{1}{xy}dy = \frac{1}{xy}dx - \frac{\ln y}{x^2}dx$$
(on dividing by x^2y^2)
$$\Rightarrow \frac{1}{xy}dx - \frac{\ln x}{y^2}dy + \frac{1}{xy}dy - \frac{\ln y}{x^2}dx = 0$$
$$\Rightarrow d\left(\frac{\ln x}{y}\right) + d\left(\frac{\ln y}{x}\right) = 0$$
On integrating both sides, we get
$x \ln x + y \ln y = cxy$.

29. (A)
$[\{f(x)\}] = 0$
$$\Rightarrow \quad e^{2x} + e^x - 2 = 0$$
$$\Rightarrow \quad (e^x + 2)(e^x - 1) = 0$$
$$\Rightarrow \quad e^x = 1 \text{ is the only solution}$$
(B) $I = \int_{-\pi}^{\pi} \sqrt{\frac{|\sin x|}{1+\tan^2 x}}\,dx = 2\int_0^{\pi} \sqrt{\frac{\sin x}{1+\tan^2 x}}\,dx$
$$= 4\int_0^{\pi/2} \sqrt{\sin x}.\cos x\,dx = \left[4.\frac{2}{3}(\sin x)^{3/2}\right]_0^{\pi/2} = \frac{8}{3}.$$
(C) $f(x) = 2\sin x|\cos x|$ is periodic with the period 2π.
Number of critical points in $[-2\pi, 2\pi]$ is twice that in $[0, 2\pi]$
$$f(x) = \begin{cases} \sin 2x; & 0 \le x \le \frac{\pi}{2} \\ -\sin 2x; & \frac{\pi}{2} \le x \le \frac{3\pi}{2} \\ \sin 2x; & \frac{3\pi}{2} \le x \le 2\pi \end{cases}$$
(D) $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1} - ax - b\right) = 0$
$$\lim_{x\to\infty} \frac{x^2+1-ax(x+1)-b(x+1)}{x+1} = 0$$
$$\lim_{x\to\infty}\left(\frac{(1-a)^2x^2-(a+b)x+1-b}{1+x}\right) = 0$$
$1 - a = 0$ *and* $a + b = 0$
$a = 1, b = -1$

30. (A) $f(x) = \ln(x^2 + x - 2)^2$
$= \ln\ ((x+2)(x-1))^2$
At $x = -2, 1$; $f(x)$ is not defined.
$\therefore$ Domain $x \in R - \{-2, 1\}$.
(B) $\lim_{x\to 1} 2x + \left(\frac{(b+1)(b^3-2b^2+2b-4)}{b^2-b-2}\right) \ge 2$
$$2 + \frac{(b+1)(b^3-2b^2+2b-4)}{b^2-b-2} \ge 2$$
$$\frac{(b+1)(b^3-2b^2+2b-4)}{b^2-b-2} \ge 0$$
$$\frac{(b+1)(b-2)(b^2+2)}{(b+1)(b-2)} \ge 0$$
$b \in R - \{-1, 2\}$.
(C) For $x \in [0,1]$ and $1 \le \frac{1}{\sqrt{1-x^4}} \le \frac{1}{\sqrt{1-x^2}}$
$$\int_0^1 dx < \int_0^1 f(x)dx < \int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

$$1 < \int_0^1 f(x)\,dx < [\sin^{-1} x]_0^1$$

$$1 < \int_0^1 f(x)\,dx < \frac{\pi}{2}$$

(D)If P (x, y) is a point on curve , then its reflection about $x + y = 0$ is $P'(-y, -x)$.

Section 4: (Integer Answer Type)

31. Given $f(x).f'(x) \geq x\sqrt{1-(f(x))^4}$

$$\frac{f(x)f'(x)}{\sqrt{1-(f(x))^4}} \geq x$$

$$\int_{x_1}^{x_2} \frac{f(x)f'(x)}{\sqrt{1-(f(x))^4}}dx \int_{x_1}^{x_2} x\,dx$$

Put $(f(x))^2 = t \Rightarrow 2f(x)f'(x) = dt$

$$\therefore \frac{1}{2}\int_{t_1}^{t_2} \frac{dt}{\sqrt{1-t^2}} \geq \int_{x_1}^{x_2} x\,dx$$

Where $t_1 = \lim_{x \to x_1^+} (f(x))^2 = 1$ and $t_2 = \lim_{x \to x_2^-} (f(x))^2 = \frac{1}{2}$

$$\frac{1}{2}\int_1^{1/2} \frac{dt}{\sqrt{1-t^2}} \geq \left[\frac{x^2}{2}\right]_{x_1}^{x_2} \Rightarrow \left[\sin^{-1} t\right]_1^{1/2} \geq x_2^{\,2} - x_1^{\,2}$$

$$x_2^{\,2} - x_1^{\,2} \leq \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3}$$

$x_2^{\,2} - x_1^{\,2} \geq \frac{\pi}{3} = 1.047$ The least integral value = 2

32. $y^2 = 4[\sqrt{y}] = 1 \Rightarrow y^2 = 4x$

Similarly for $x \in [1,4), [\sqrt{x}] = 1$ and $x^2 - 4[\sqrt{x}]y$ would transform into $x^2 = 4y$

Required area is being shaded

$$= \int_1^2 (2\sqrt{x} - 1)dx + \int_2^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$$

$= \left(\frac{4}{3}x^{3/2} - x\right)_1^2 + \left(\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right)_2^4 = \frac{11}{3}$. Therefore, 12A/11 = 4

33. Pair of lines $y^2 - 3y + 2 = 0 \Rightarrow$ lines are y= 2, y= 1,

Let [a] = b.

Now curves are $y = bx^2$ and $y = \frac{b}{2}x^2$

Bounded area

$$= 2\left[\int_1^2 (x_2 - x_1)\,dy\right]$$

$$= 2\left[\int_1^2 \left(\sqrt{\frac{2y}{b}} - \sqrt{\frac{y}{b}}\right)dy\right] = 2\left[\sqrt{\frac{2}{b}}.\frac{y^{3/2}}{3/2} - \frac{1}{\sqrt{b}}\frac{y^{3/2}}{3/2}\right]_1^2$$

$$= \frac{4}{3\sqrt{b}}[(\sqrt{2}-1)y^{3/2}]_1^2 = \frac{4}{3\sqrt{b}}(\sqrt{2}-1)(2\sqrt{2}-1)$$

Area will be maximum when b = [a] is atleast as

$a \geq 1 \Rightarrow [a]_{least} = 1 \Rightarrow 1 \leq a < 2.$

34. Ans. : 4

The region is a square bounded by the lines x = -2, x = 2, y = -2, y = 2

35. Ans. : 2 We have,

$$f(x) = x + x^2\int_0^1 z\,f(z)\,dz + x\int_0^1 z^2\,f(z)\,dz = x + x^2\lambda_1 + x\lambda_2 \qquad ...(1)$$

where $\lambda_1 = \int_0^1 z\,f(z)\,dz$ and $\lambda_2 = \int_0^1 z^2\,f(z)\,dz$

Here, $\lambda_1 = \int_0^1 z\,f(z)\,dz = \int_0^1 z\{z + z^2\lambda_1 + z\lambda_2\}\,dz$

$= \int_0^1 z\{(1+\lambda_2)z + z^2\lambda_1\}\, dz = (1+\lambda_2)\left(\frac{z^3}{3}\right)_0^1 + \lambda_1\left(\frac{z^4}{4}\right)_0^1 = \frac{1+\lambda_2}{3} + \frac{\lambda_1}{4}$

$\Rightarrow 9\lambda_1 - 4\lambda_2 = 4$ ...(2)

Also, $\lambda_2 = \int_0^1 z^2 f(z)\,dz = \int_0^1 \{(1+\lambda_2)z^3 + z^4\lambda_1\}dz = \frac{(1+\lambda_2)}{4} + \frac{\lambda_1}{5}$

$\Rightarrow 15\lambda_2 - 4\lambda_1 = 5$ ...(3)

On solving equations (2) and (3), we get $\lambda_1 = \frac{80}{119}, \lambda_2 = \frac{61}{119}$

$\therefore$ From equation (1), $f(x) = x + \frac{80}{119}x^2 + \frac{61}{119}x = \frac{20x}{119}(4+9x).$

36, Ans. : 3

Sol. $x = 4t^2, y = 4t^3$. $\frac{dy}{dx} = \frac{12t^2}{8t} = \frac{3t}{2}$

Any line through origin is y = mx.

Solving with the curve, $\not{4}\, t^3 = m \times \not{4}\, t^2 \Rightarrow m = t$

$\therefore$ Equation of tangent at P is $y - 4m^3 = \frac{3m}{2}(x - 4m^2)$, and

tangent Q is $y + \frac{4}{m^3} = \frac{-3}{2m}\left(x - \frac{4}{m^2}\right)$.

Eliminate m to get the locus as $4y^2 = 4(3x - 4) \Rightarrow y^2 = 3(x - 4/3)$

Latus rectum = 3.

37. Ans. : 4

Let $y = \lim_{x\to 0^+} \frac{\log_{\sin x}\cos x}{\log_{\sin(x/2)}\cos(x/2)} = \lim_{x\to 0^+} \frac{\ln\cos x \, \ln\sin(x/2)}{\ln\cos(x/2)\ln\sin x}$

$= \left\{\lim_{x\to 0^+} \frac{\ln\cos x}{\ln\cos(x/2)}\right\} \text{x} \left\{\lim_{x\to 0^+} \frac{\ln\sin(x/2)}{\ln\sin x}\right\} = y_1 \text{ x } y_2.$

Using L'Hospital's Rule,

$y_1 = \lim_{x\to 0^+} \frac{\sin x.\cos(x/2)}{(\cos x).\sin(x/2)(1/2)} = 4$

$y_2 = \lim_{x\to 0^+} \frac{\ln\sin(x/2)}{\ln\sin x} = \lim_{x\to 0^+} \frac{\frac{1}{2}\cos\frac{x}{2}}{\sin\frac{x}{2}} \text{x} \frac{\sin x}{\cos x} = 1$

Required limit = $= y = y_1 \text{ x } y_2 = 4\text{x}1 = 4$.

38. Ans. : 8

Sol. The circle is $x^2 + y^2 = 16$.

Let l be the latus rectum of the parabola then its equation is $y^2 = l(x + 4)$.

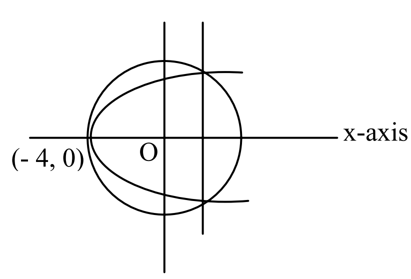
Any point on the circle is (4 cos θ , 4 sin θ)

But it lies on the parabola.

$\therefore l = 2a \sin^2 \theta/2$.

Required area = A = $2\int_{-4}^{4\cos\theta} \sqrt{l(x+4)}\, dx$

$= \frac{16\times 16}{3} \sin\frac{\theta}{2} \cos^3\frac{\theta}{2}$



$= \frac{32}{3}(2\sin\theta + \sin 2\theta)$

$\frac{dA}{d\theta} = 0 \quad \Rightarrow \theta = \pi/3 \text{ or } \pi,$

$\frac{d^2A}{d\theta^2} = \frac{64}{3}(\sin\theta - 2\sin 2\theta) < 0 \text{ if } \theta = \pi/3.$

$\therefore$ A is maximum for $\theta = \pi/3$.

$A = \frac{16\times 16}{3}\sin\frac{\pi}{6} . \cos^3\frac{\pi}{6} = \frac{16\times \not{16}}{\not{3}} \times \frac{1}{\not{2}} \times \frac{\not{3}\sqrt{3}}{\not{8}} = 16\sqrt{3}.$

$\therefore \frac{A}{2\sqrt{3}} = \frac{16\sqrt{3}}{2\sqrt{3}} = 8.$

PHYSICS : ANSWERS AND SOLUTIONS :

39. Ans. : C

Sol. $x_B = l\sin\theta$, $v_0 = l\cos\theta\dfrac{d\theta}{dt}$ $\qquad \dfrac{d\theta}{dt} = \dfrac{v_0}{l\cos\theta}$

$x_B^2 + y_A^2 = l^2$, $\dfrac{dy_\theta}{dt} = -v_0\tan\theta$

Speed of the centre of mass = $\dfrac{v_0}{2}\sec\theta$

$$\therefore U_K = \frac{1}{2}\left(\frac{v_0}{2}\sec\theta\right)^2 + \frac{1}{2}.\frac{ml^2}{l^2}\omega^2 = \frac{2}{3}mv_0^2.$$

40. Ans. : A

Sol. For equilibrium mg(2r) = Mg r sin θ

Mg sin θ - mg – f = 0

f = mg, Normal reaction = Mg cos θ

$\therefore \mu \ge \dfrac{m}{M\cos\theta} = \dfrac{1}{2}\tan\theta \qquad \therefore \mu \ge \dfrac{\sqrt{3}}{2}$

41. Ans. : D

Sol. $T = \dfrac{m}{4}\left(\dfrac{5l}{8}\right)\omega^2$.

42. Ans. : C

Sol. $mv_0(4R_e) = mvR_e \qquad v = 4v_0$

$$-\frac{GmM_e}{4R_e} + \frac{1}{2}mv_0^2 = -\frac{GmM_e}{R_e} + \frac{1}{2}mv^2$$

$g = \dfrac{Gm_e}{16R_e^2} \qquad \therefore v_0 = \sqrt{1.6gR_e}$

43. Ans. : ABCD

Sol. $\dfrac{m}{2}v_0\left(\dfrac{2R}{3}\right) = \left(\dfrac{4}{3}mR^2\right)\omega. \qquad \omega = \dfrac{v_0}{4R}$

$\dfrac{m}{2}v_0 = \dfrac{3m}{2}v_G \qquad \overline{v_G} = \dfrac{v_0}{3}\hat{i}$

$\overline{V_C} = v_G\hat{i} - \dfrac{R\omega}{3}\hat{i} = \dfrac{v_0}{4}\hat{i}$

Energy dissipated = $\dfrac{1}{2}\left(\dfrac{m}{2}\right)v_0^2 - \left[\dfrac{1}{2}\left(\dfrac{3m}{2}\right)\dfrac{v_0^2}{9} + \dfrac{1}{2}\left(\dfrac{4mR^2}{3}\right)\omega^2\right] = \dfrac{mv_0^2}{18}$.

44. Ans. : CD

Sol. $\dfrac{x_B}{\sin\theta} = \dfrac{x_A}{\sin(120+\theta)} = \dfrac{L}{\sin 120}$

$x_A = \dfrac{2L}{\sqrt{3}}\sin(120+\theta) \qquad \dfrac{dx_A}{dt} = \dfrac{2L}{\sqrt{3}}\cos(120+\theta)\dfrac{d\theta}{dt} = v_0$

At $\theta = 30^\circ$ $\dfrac{d\theta}{dt} = -\dfrac{v}{L}$

$x_B = \dfrac{2L}{\sqrt{3}}\sin\theta \qquad \dfrac{dx_B}{dt} = \dfrac{2L}{\sqrt{3}}\cos\theta\left(\dfrac{d\theta}{dt}\right) = -v_0$.

45. Ans. : BCD

Sol. f = mg, N = 3mg, $\qquad$ μ ≥ 1/3 for static friction.

∴ Between B and S $\qquad$ friction = ¼ (3mg)

backward on B. $\qquad$ ∴ mg – 3mg/4 = 3ma.

a = g/12, f' = mg/12, $\qquad$ N′ = mg, $\qquad$ μ′ ≥ 1/12

Given μ′ = 0.1 $\qquad a_A = a_B = \dfrac{g}{12}\hat{i}$.

46. Ans. : BCD

Sol. $4T = ma_B \qquad mg - T = ma_A$

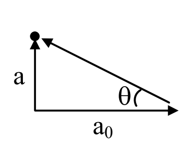
$a_A = 4a_B$.

47. Ans. : BCD

Sol. $a = a_1\tan\theta$

For on C = $ma_0\tan\theta$

$2N\cos\theta - mg = ma$



$N = \dfrac{mg + ma_0 \tan\theta}{2\cos\theta}$

48. Ans. : A – R; B – S; C – P, S; D – Q

Sol. At $r = \dfrac{3R_e}{2}$ orbital velocity = $\sqrt{\dfrac{2GM_e}{3R_e}}$;

escape velocity = $\sqrt{\dfrac{4GM_e}{3R_e}}$

when $v_0 = \sqrt{\dfrac{8GM_e}{15R_e}}$ the object just clears the earth;

when $v_0 = \sqrt{\dfrac{GM_e}{2R_e}}$ it falls on earth's surface.

49. Ans. : A – P, B – P, C – P, S; D – Q, R.

50. Ans. : 1

Sol. $F\left(\dfrac{l}{2}\right) - 5\left(\dfrac{l}{2} - \dfrac{1}{5}\right) = 0$; $5 - F = 1(2)$ $F = 3$.

$\therefore\ l = 1$ m; $\therefore$ [1]

51. Ans. : 2

Sol. $m_1 = M\dfrac{(R/2)^2}{R^2 - (R/2)^2} = \dfrac{M}{3}$, $m_2 = M\dfrac{R^2}{R^2 - (R/2)^2} = \dfrac{4M}{3}$

Centre of mass is at a distance $\dfrac{R}{6}$

$I_1 = \dfrac{4M}{3}\left(\dfrac{R^2}{2}\right) + \dfrac{4M}{3}\left(\dfrac{R}{6}\right)^2 = \dfrac{19MR^2}{27}$

$I_2 = \dfrac{\left(\dfrac{M}{3}\right)\left(\dfrac{R}{2}\right)^2}{2} + \dfrac{M}{3}\left(\dfrac{2R}{3}\right)^2 = MR^2\left(\dfrac{4}{27} + \dfrac{1}{24}\right)$.

$\therefore\ I_G = I_1 - I_2 = \dfrac{37MR^2}{72} = \dfrac{37MR^2}{(36)2}$ [2]

52. Ans. : 7

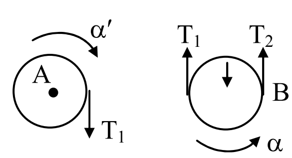
Sol. $T_2r - T_1r = \dfrac{mr^2}{2}\alpha$

$mg - T_1 - T_2 = ma_1$

$a_1 = r\alpha$

$T_1r = \dfrac{(3m)r^2}{2}\alpha'$ $T_1 = \dfrac{3m}{2}r\alpha'$

$r\alpha' - 2r\alpha$ $\therefore\ T_2 = \dfrac{7mg}{15}$ [7]



53. Ans. : 9

Sol. $M = \displaystyle\int_0^R \rho_0 \frac{r}{R}\left(4\pi r^2 dr\right) = \pi\rho_0 R^3$.

$I_{dai} = \displaystyle\int_0^R \frac{2}{3}\left(4\pi r^2 dr\right)\rho \cdot r^2 = \frac{13MR^2}{9}$ $\therefore$ [9]

54. Ans. : 4

Sol. $M = \displaystyle\int_{-l/2}^{+l/2} \rho_0\left(1 - \frac{x^3}{2l^3}\right)dx = \rho_0 l$

$x_G = \dfrac{1}{3}\displaystyle\int_{-l/2}^{+l/2} \rho_0 x\left(1 - \frac{x^3}{2l^3}\right)dx = -\frac{l}{160}$; $\therefore$ [4]

55. Ans. : 9

Sol. $\frac{3}{4}\, l\omega = v_0$; $\overline{L_0} = I_0\omega = \left(\frac{17Ml^2}{12}\right)\omega = \frac{17Mv_0 l}{9}$; $\therefore$ [9]

56. Ans. : 5

Sol. $\frac{1}{2}\, mv^2 = mg\, R\frac{\sqrt{3}}{2}$; $v^2 = \sqrt{3}\, gR$

$N - mg\, \frac{\sqrt{3}}{2} = \frac{mv^2}{R}$; N = Normal reaction of A on B = $\frac{3\sqrt{3}}{2}$ mg.

$N_2 - \frac{mg}{4} - \frac{3\sqrt{3}mg}{2}\, (\cos 30^3) = 0$. $N_2 = \frac{5mg}{2}$; $\therefore$ n = 5 [5]

57. Ans. : 8.

Sol. $\frac{a_0}{x'} = \frac{2a_0}{x_0 - x'}$; $x' = \frac{x_0}{3}$.

The area under a – x graph $= \frac{1}{2}\, a_0\left(\frac{x_0}{3}\right) - \frac{1}{2}\left(\frac{x_0}{2} - \frac{x_0}{3}\right)\left(\frac{a_0}{2}\right) = \frac{1}{2}\, v^2 - 0$.

$\therefore$ kinetic energy = $\frac{1}{2}\, mv^2 = \frac{ma_0 x_0}{8}$. $\therefore$ [8]