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IIT ACADEMY

MODEL TEST FOR IIT –JEE 2010

UNIT-1: PAPER-I

04-01-2010 – IIT-JEE- UNIT – 1 – PAPER – 1 – ANSWERS :

CHEMISTRY : 1) B, 2) C, 3) D, 4) B, 5) B, 6) D, 7) A, 8) C,
9) AC, 10) ABD, 11) ABCD, 12) BD
13) D, 14) A, 15) B, 16) B, 17) D, 18) D
19) A – RST, B – RS, C – P, Q – PQ; 20) A – QRT, B – RT, C – PQR, D – PQ.
MATHEMATICS : 21) D, 22) B, 23) A, 24) A, 25) C, 26) B, 27) B, 28) D
29) AD, 30) AC, 31) B, 32) ACD
33) C, 34) D, 35) C, 36) A, 37) C, 38) B,
39) A – R, B – P, C – S, D – Q; 40) A – PQR, B – P, C – S, D – PR
PHYSICS : 41) A, 42) B, 43) B, 44) A, 45) C, 46) C, 47) C, 48) D
49) BCD, 50) ABCD, 51) A, 52) BCD
53) C, 54) B, 55) B, 56) A, 57) D, 58) C
59) A – S, B – R, C – S, D – P, 60) A – Q, B – P, C – R, D – R.

SOLUTIONS – CHEMISTRY :

1. (B) $2Mg + O_2 \rightarrow 2MgO$,

Before reaction $\frac{2}{24}$ - $\frac{1}{32}$

After reaction $\left[\frac{1}{12}-\frac{1}{16}\right]$ $-$ $\frac{1}{16}$

$\frac{1}{48}$ $\frac{1}{16}$

$\because$ Mg and MgO both reacts with HCI

$Mg + 2HCl \longrightarrow MgCl_2 + H_2$

$\frac{1}{48}$

$MgO + 2HCl \longrightarrow MgCl_2 + H_2O$

$\frac{1}{16}$

total moles of HCI = $\frac{1}{24}+\frac{1}{8}$, required = $\frac{1}{6}$

volume of HCI = $\frac{0.166}{0.02} = 8.33\text{Litre}$

2. (C) Work function of $Li^{+2} = E_\infty - E_{ground} = 0-\left[-E\times\frac{3^2}{1^2}\right] = 9E$

$E_P = W + \frac{1}{2}mv^2$ or $E_p = 9E + \frac{1}{2}mv^2$

$v = \sqrt{\frac{2(E_p - 9E)}{m}}$

3. (D) For H_2 and He gas inversion temperature is very less and hence show heating during Joul-. Thomson experiment.

4. (B) Total no. of moles of cation substituted per mole of $K_4[Fe(CN)_6] = 3$

5. (B) Bond order for NO^+ is highest i.e., 3, hence bond length is shortest

6. (D) According to Fajan's rule

7. (A) $\frac{1}{2}mv^2 = \frac{k(Zq_1)q_2}{r}$

$\frac{q_2}{m} = \frac{rv^2}{2k_1.q_1.Z} \Rightarrow \frac{2.5\times10^{-14}\times(2.1\times10^7)^2}{2\times9\times10^9\times79\times1.6\times10^{-19}}$ = 4.84 × 10^7 coloumb/gm

8. (C) At low pressure

$\left(P+\frac{a}{V^2}\right)V = RT$ or $PV^2 - RTV + a = 0$

$V = \frac{RT \pm \sqrt{R^2T^2 - 4Pa}}{2P}$ $\quad [\because 4Pa = R^2T^2]$

9. (A,C)

10. (A,B,D)

11. (A,B,C,D)

12. (B,D) Normality = $\frac{V}{5.6} \Rightarrow \frac{20}{5.6} = 3.528N$

13. (D)

14. (A)

15. (B) Molarity of $NaCO_3$ = 10 M_1

Molarity of $NaHCO_3$ = 10 M_2

In part – A

For first ppt. $NaCO_3 + CaCI_2 \rightarrow CaCO_3 + 2NaCI$

No. of moles of Na_2CO_3 = No. of moles of $CaCO_3$

$10M_1 \times \frac{50}{1000} = \frac{w_1}{100}$

$w_1 = 50\ M_1$

For second part

$NaHCO_3 + CaO \rightarrow CaCO_3 + NaOH$

No. of moles of $NaHCO_3$ = No. of moles of $CaCO_3$

$10M_2 \times \frac{50}{1000} = \frac{w_2}{100}$

$w_2 = 50\ M_2$

In part – B

On boiling $2NaHCO_3 \rightarrow Na_2CO_3 + H_2O + CO_2$

$Na_2CO_3 + CaCI_2 \rightarrow CaCO_3 + 2NaCI$

No. of moles of Na_2CO_3 = No. of moles of $CaCO_3$

$$\left[\frac{10M_2 \times \frac{50}{1000}}{2}\right] + 10M_1 \times \frac{50}{1000} = \frac{w_3}{100}$$

16. (B) $P_{final} = \frac{P_{H_2} \times V_{initial} + P_{N_2} \times V_{initial}}{V_{final}}$

$= \frac{4.1 \times 6 + 0.82 \times 10}{6+10} = 2.05atm$

17. (D) Moles of He = $\frac{PV}{RT} \Rightarrow \frac{4.1 \times 6}{0.082 \times 300} = 1$

Moles of N_2 = $\frac{PV}{RT} \Rightarrow \frac{0.82 \times 10}{0.082 \times 300} = \frac{1}{3}$

Total kinetic energy = $\frac{3}{2}nRT = \frac{3}{2}\left(1+\frac{1}{3}\right) \times 8.314 \times 300$

18. (D) Partial pressure of N_2 = $\frac{1/3}{1+1/3} \times P_{total} \Rightarrow \frac{1}{4} \times 1 = 0.25atm$

19. A – RST, B – RS, C – P, D – PQ.

20. A – QRT, B – RT, C – PQR, D – PQ.

SOLUTIONS : MATHEMATICS :

21. Ans. : D

$$f'(x) = \underset{h\to 0}{Lt} \frac{f(x)\left\{\frac{f(x+h)}{f(x)} - 1\right\}}{h}$$

$= f(x) . \underset{h\to 0}{Lt} \frac{f(h)-1}{h}$

$= f(x) . f'(0) \quad (\because f(0) = 1)$

$= a\ f(x) \Rightarrow f(x) = e^{ax}$.

22. Ans. : B

$f^2(x) = f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{-1}{x}$

$f^3(x) = f(f^2(x)) = f\left(\frac{-1}{x}\right) = \frac{x+1}{1-x}$

and $f^4(x) = f(f^3(x) = f\left(\frac{x+1}{1-x}\right) = x$

$f^{2010}(x) = f^2(f^{2008}(x)) = f^2(x) = \frac{-1}{x} = g(x).$

$\therefore \int_{1/e}^{e} g(x)dx = -\int_{1/e}^{1} \frac{1}{x}dx = -1.$

23. Ans. : A

$\underset{n\to\infty}{Lt}\left(\cos\frac{x}{n}\right)^n = e^{\underset{n\to\infty}{Lt}\left(\cos\frac{x}{n}-1\right).n}$

$= e^{\underset{n\to\infty}{Lt} -\left(2\sin^2\frac{x}{2n}\right).n} = e^{-\underset{n\to\infty}{Lt}\left(\sin\frac{x}{2n}\right)^2.\frac{x^2}{2n}}$

$= e^{-1\times 0} = e^0 = 1.$

24. Ans. : A

The equation of the parabolas will be of the form $y^2 = 4a(x - a)$ which involves only one parameter ‘a’.

25. Ans. : C

f(x) is defined if $2 - \left[\log_{\sqrt{2}}\left(16\sin^2 x+1\right)\right] > 0$

If $\left[\log_{\sqrt{2}}\left(16\sin^2 x+1\right)\right] < 2$

If $\left[\log_{\sqrt{2}}\left(16\sin^2 x+1\right)\right] < 2$

If $16\sin^2 x+1 < 2$, we know $16\sin^2 x + 1 \geq 1 \ \forall \ x \in R$

$\therefore 1 \leq 16\sin^2 x + 1 < 2$

$\Rightarrow 0 \leq \log_{\sqrt{2}}\left(16\sin^2 x+1\right) < 2$

$\Rightarrow \left[\log_{\sqrt{2}}\left(16\sin^2 x+1\right)\right] = 0, 1.$

26. Ans. : B

We know, if $I = \int_a^b f(x)dx$ then

$I = \frac{1}{2}\int_a^b \{f(x)+f(a+b-x\}dx$. Using this

$\int_{-5}^{2} \frac{3x^2+x-3}{x^2+3x+3}dx = \frac{1}{2}\int_{-5}^{2} \frac{6x^2+18x-18}{x^3+3x+3}dx = 21$

27. Ans. : B

If f(x) is continuous at x = a, whether a is rational or irrational, whether a^+ and a^- are rational or irrational. The value of the function should be same as its limits. This happens only if $\sin^2 a = -\sin^2 a$ or $2\sin^2 a = 0$ or $\sin a = 0$ or $a = n\pi$.

28. Ans. : D

$g'(x) = f'\{(\tan x - 1)^2 + 3\}\ 2(\tan x - 1)\sec^2 x$. Since $f''(x) > 0$.

$f'(x)$ is increasing. So $f'\{(\tan x - 1)^2 + 3\} > f'(3)$ for $x \in \left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{4},\frac{\pi}{2}\right)$,

also $\tan x - 1 > 0 \ \forall \ x \in \left(\frac{\pi}{4},\frac{\pi}{2}\right)$.

$\therefore$ g(x) is increasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

29. Ans. : AD

$$f(x)^2 = \left.\begin{array}{ll} x^2 & \text{when } x > 0 \\ 0 & \text{when } x = 0 \\ -x^2 & \text{when } x < 0 \end{array}\right\} \qquad f'(x) = \begin{array}{ll} 2x & \text{when } x \geq 0 \\ 2x & \text{when } x < 0 \end{array}$$

It is continuous and differentiable on R. Also $f'(x) \geq 0 \ \forall\ x$. Hence it is monotonically increasing. So it is bijective.

30. Ans. : AC

Let the curve by y = f(x) then given that sub-tangent length = 2x $\Rightarrow \dfrac{y}{\frac{dy}{dx}} = 2x$.

$$\frac{dy}{y} = \frac{1}{2}\frac{dx}{x}$$

$\Rightarrow \log y = ½ \log x + \log c.$

$\therefore\ y = c\sqrt{x}$ or $y^2 = c^2x$, substituting (1, 2)

$4 = c^2$ or $c = 2$

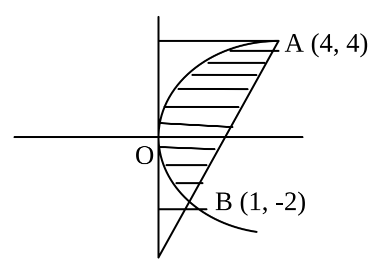
$\therefore$ curve is $y^2 = 4x$, a parabola.

Solving $y^2 = 4x$ and $2x – y – 4 = 0$, we get

$4(x - 2)^2 = 4x$ or $x^2 – 4x + 4 = x$

$2^2 – 5x + 4 = 0 \qquad (x – 4)(x – 1) = 0$

$x = 1, 4, \quad y = –2, 4$



Required area

$$= \int_{-2}^{4}\left\{\left(\frac{y+4}{2}\right) - \frac{y^2}{4}\right\}dy = 9$$

differentiating $y^2 = 4x$, w.r.to x, we get

$$2y\ .\ \frac{dy}{dx} = 4 \text{ or } \frac{dy}{dx} = \frac{4}{2y} = \frac{1}{2} \text{ at A and } -1 \text{ at B.}$$

If the line is a normal, if should be a normal at A or B. Then $\dfrac{-1}{\frac{dy}{dx}} = 2$ at A or B which is not correct.

$\therefore$ The line is not a normal.

31. Ans. : B.

Let d(t) be the distance from Hyderabad on the interval [0, 2]. Then d(t) is differentiable because the velocity exists. Using mean value theorem, there exist $C \in [0, 2]$ such that

$$\frac{d(2) - d(0)}{2 - 0} = d'(C) \Rightarrow d(2) = d(0) + 2\,d'(C).$$

$\therefore\ d(2) = 20 + 2\,d'(C)$

$d'(C)$ is velocity at time t = C and hence $60 \leq d'(C) \leq 70$

$\Rightarrow 140 \leq d(20 \leq 160$..

32. Ans. : ACD

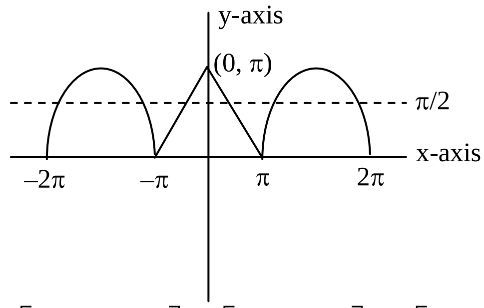
Let $f(x) = \pi – |x| \quad$ when $|x| \leq \pi$

$= \pi\,|\sin x| \quad$ when $\pi < |x| \leq 2\pi$

and $g(x) = 1$ for $\dfrac{-\pi}{2} < x \leq \pi/2$

$= 2$ for $\dfrac{\pi}{2} < x \leq \pi$. Then

33. Ans. : C



g o f(x) = 1, for $x \in \left[-2\pi, \frac{-11\pi}{6}\right] \cup \left[\frac{-7\pi}{6}, \frac{-\pi}{2}\right] \cup \left[\frac{\pi}{2}, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$

= 2, for $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right) \cup \left(\frac{-11\pi}{6}, \frac{-7\pi}{6}\right)$

34. Ans. : D.

35. Ans. : C

36. Ans. : A

37. Ans. : C

38. Ans. : B

Given f(xy) = f(x) . f(y). Differentiating partially w.r. to x and y and eliminating f(xy) term, we get $\frac{xf'(x)}{f(x)} = \frac{yf'(y)}{f(y)} \Rightarrow \frac{x.f'(x)}{f(x)}$ = constant.

On solving we get $f(x) = x^2$. $\therefore$ C_1 is $y = x^2$, a parabolic

For C_2 it is given $\frac{dy}{dx} = -xy^2$.

$\Rightarrow y^{-2} . dy = -x\,dx$ or $\frac{-1}{y} = \frac{-x^2}{2} + C = \frac{-x^2 + 2C}{2}$.

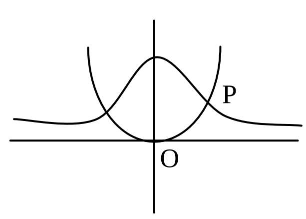
$\therefore y = \frac{2}{-2C + x^2}$. substituting (1, 1).

$1 = \frac{2}{-2C+1} \Rightarrow -2C + 1 = 2 \Rightarrow C = \frac{-1}{2}$

$\therefore$ C_2 is $y = \frac{2}{x^2+1}$. C_1 and C_2 intersect at C(±1, 1) $\therefore$ P is (1, 1)

Sub-normal to C_2 at P is $\left|y\frac{dy}{dx}\right|$ = 1 and subtantut

to C_1 at P is $\frac{y}{\frac{dy}{dx}} = \frac{1}{2}$



Area = $2\int_0^1 \left(\frac{2}{1+x^2} - x^2\right) dx = \pi - \frac{2}{3}$.

39. Ans. : A → R, B → P, C → S, D → Q

(A) Period of sin 36x is $\frac{\pi}{18}$, that of tan 42x is $\frac{\pi}{42}$

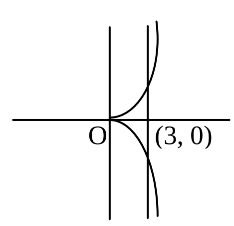
$\therefore$ Period of f(x) = p = π/6.

(B) $y^2 = a^2(x - a) \Rightarrow a^2x - a^3 = y^2$. Differentiating w.r. to a.

$2ax - 3a^2 = 0$. Eliminating a, we get $27y^2 = 4x^3$ as the curve.

Required area = $2\int_0^3 y\,dx$

$= \frac{24}{5}$ sq. units.



(C) tangent will be parallel to the line joining given points. Using Legranges the theorem. f′(c) = slope of the line

$\therefore 3x^2 = \frac{8+1}{2+1} = 3 \Rightarrow x = 1, y = 1.$

(D) The angel of intersection is given by tanθ = 5/3.

40. Ans. : A → PQR, B → P, C → S, D → PR.

Let $-2 \le x \le 0$ and $-2 \le t \le x$ then

$g(x) = \int_{-2}^{x} f(t)\,dt = \int_{-2}^{x} f(-1)\,dt = -(x+2).$

Let $0 < x \le 1$ and $-2 \le t \le x$ then

$g(x) = \int_{-2}^{x} f(t)\,dt + \int_{0}^{x} f(t)\,dt = \int_{-2}^{0} (-1)\,dt + \int_{0}^{x} -(t-1)\,dt$

$= \dfrac{-3-(x-1)^2}{2}$. Easily we can observe that g(x) is continuous at x = 0, not differentiable at x = 0 and has a local minimum at x = 0 because $g'(x) < 0$ for $x < 0$ and $g'(x) > 0$ for $x > 0$

SOLUTIONS : PHYSICS :

41. Ans. : A

Sol. : $m(V_1 - V_2) = \frac{1}{2}mv^2 - 0$

$V_1 = -\frac{GM}{6R}$, $V_2 = -\frac{GM}{R}$, $v = \sqrt{\frac{5GM}{3R}}$.

42. Ans. : B

Sol. : $mv_0R_e = mv(h + R_e)$

$\frac{1}{2}mv_0^2 - \frac{GMM_e}{R_e} = \frac{1}{2}mv^2 - \frac{GMM_e}{R_e + h}$

$v_0^2 = \frac{3GM_e}{R_e}$ solving for h. $h = 2R_e$.

43. Ans. : B

Sol. : $N = Mg + \frac{F}{\sqrt{2}}$, $f = \frac{F}{\sqrt{2}}$

For static friction $\mu_S \ge \frac{f}{N}$

$F(\sqrt{3}-1) \ge -\sqrt{6}Mg$ which is satisfied for any value of F.

$\therefore f = \frac{F}{\sqrt{2}}$ always.

44. Ans. : A

Sol. : $ma_p(R) = \frac{5}{3}mp^2\alpha$

$ma_p - f = ma$

$mg - f = 3ma_p$

$a = \frac{3g}{17}$; $\alpha = \frac{3g}{17R}$

45. Ans. : C

Sol. : $v_0 = 2r\omega$ U_K ring $= \frac{1}{2}\left(2mr^2\right)\omega^2 = \frac{mv_0^2}{4}$

U_K system $= \frac{1}{2}mv_0^2 + \frac{mv_0^2}{4} = \frac{3}{4}mv_0^2$.

46. Ans. : C

Sol. : $mg\sin\theta - 3f = Ma$

$f(2r) = \frac{3}{2}mr^2\alpha$ $a = 2r\alpha$.

$a = \dfrac{g\sin\theta}{1+\frac{9m}{8M}}$ $\therefore$ acceleration of the cylinders $= \frac{a}{2} = \dfrac{g\sin\theta}{2+\frac{9m}{4M}}$

47. Ans. : C

Sol. : Conservation of angular momentum about the centre of mass.

$mv_0\frac{l}{4}\sin 30 = \left(m\frac{l^2}{16} + m\frac{l^2}{12} + m\frac{l^2}{16}\right)\omega$.

$\omega = \frac{3v_0}{5l}$

48. Ans. : D

Sol. : N = 2mg cos 60 + mg = 2mg

F (2r sin 60) = $\frac{3}{2}mr^2\alpha$, $mr\alpha = \frac{2F}{\sqrt{3}}$

$\frac{\sqrt{3}}{2}F + f$ = mrα F = $\frac{2F}{\sqrt{3}} - \frac{\sqrt{3}F}{2}$ = $\frac{F}{2\sqrt{3}}$

$\therefore \mu \geq \frac{|f|}{N} = \frac{2mg}{2\sqrt{3}} \Big/ 2mg = \frac{1}{2\sqrt{3}}$.

49. Ans. : BCD

Sol. : Work done by gravity = $U_{g_1} - U_{g_2}$

= 10mgr – 2mgr = 8mgr

Kinetic energy = 8mgr.

N + mg = $\frac{mv^2}{r}$ = 16mg, N = 15mg

at M, N′ = $\frac{mv'^2}{r}$ = 18mg.

50. Ans. : ABCD

Sol. : Vertical acceleration = 0

At t = 0; $a_C = 0$; $a_t = r\alpha$ F = m r α

$\omega^2 = 2\alpha(2\pi) = 4\pi\alpha$; a = r α $\sqrt{1+16\pi^2}$.

51. Ans. : A

Sol. : $a_B = a_0 \sec\theta - g\tan\theta > 0$ $a_0 > g\sin\theta$.

52. Ans. : BCD

Sol. : $v^2 = (u\cos\alpha)^2 + (u\sin\alpha - gt)^2$

Work done by gravity from P to Q = 0

Average power = 0

At P power = 0.

53. Ans. : C

54. Ans. : B

55. Ans. : B

Sol. 53, 54, 55 :

V_1 = velocity of the rod, V_2 = velocity of he particle,

W = angular velocity of the rod. $V_1 = l\omega/2$.

$2Mv_1 + Mv_2 = Mv_0$ ……… (1)

(impulse by hinge = 0)

$\left(\frac{2ml^2}{3}\right)\omega + Mv_2x = Mv_0x$ …… (2)

$\frac{x\omega - v_2}{v_0} = e$ …… (3)

Solving x = $\frac{2l}{3}$, $\omega = \frac{3v_0(1+e)}{5l}$.

$v_2 = \frac{v_0(2-3e)}{5}$.

56. Ans. : A

Sol. : x = R sec θ, x′ = (R sec θ)θ′

= Rω sec θ tan θ.

57. Ans. : D

Sol. : x″ = $\frac{R\omega^2(1+\sin^2\theta)}{\cos^3\theta}$ = $14R\omega^2$ (θ = 60°)

58. Ans. : C

Sol. : x″ = $\frac{10R\omega^2}{3\sqrt{3}}$ at θ = 30°

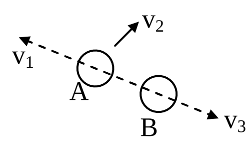
N cos θ = Mx″

N = $\frac{20}{9}$ $MR\omega^2$.

59. Ans. : A – S, B – R, C – S, D – P

Sol. : $2mv_3 - mv_1 = mv_0 \cos 30$

$mv_2 = mv_0 \sin 30$



$$\frac{v_1 + v_3}{v_0 \cos 30} = \frac{1}{2}$$

$$v_1 = 0,\ v_2 = \frac{v_0}{2},\ v_3 = \sqrt{3}\frac{v_0}{4}$$

$$U_{KA} = \frac{mv_0^2}{8},\quad U_{KB} = \frac{3mv_0^2}{16},\quad U_{K\,system} = \frac{5mv_0^2}{16}$$

$$\therefore \text{ Energy dissipated} = \frac{3mv_0^2}{16}.$$

60. Ans. : A – Q, B – P, C – R, D – R

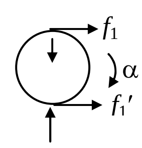
Sol. : a_P = acceleration of plank.

$$R\alpha = \frac{a_p}{2} \qquad f_1 + f_1' = \frac{ma_p}{2}$$

$$f_1(2r) = \left(\frac{3}{2}mr^2\right)\alpha$$

$$f_1 = \frac{3}{8}ma_p$$

$$\therefore a_P = \frac{F}{M + \frac{9m}{8}}, \qquad \text{Force on plank} = Ma_P$$



$$\text{Force on A} = \frac{ma_P}{2}$$

Friction on plank = $3f_1$.